1/fα-type fluctuation of electron transmission in one-dimensional disordered systems

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Abstract

A universal feature of 1/fα-type fluctuation is numerically observed in the system-size n dependence of the transmission amplitude t₀ in various one-dimensional disordered systems. The power spectrum P(f) of the transmission coefficient T(n) = |t₀|² exhibits the power law of 1/f², irrespective to the type of disorder of the system whether it is of short-range or of long-range correlation. That of the phase θt(n) of t₀ also does the universal power law of 1/f.4. © 2001 Elsevier Science B.V. All rights reserved.

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The electronic and phononic property of disordered systems has been receiving an enormous attention in this half century because of its huge unknown variety coming from the huge degrees of freedom of the disordered system [1].

The localization problem of the electronic wave function (w.f.) seems to be summarized by the recent argument based on the scaling theory [2]. The scaling theory claims that the localization characteristics of the w.f. is classified by only its dimensionality and symmetry of the system, and hence that the details (i.e., the variety) of disordered systems do not affect the localization characteristics. In the scaling argument, however, we only dealt with the scaling property of the “ensemble-averaged” conductance ⟨G⟩.

The “ensemble-averaged” quantity is the first moment of the distribution law over the samples and as a result leaves a lot of information in the higher moments unused.

We studied recently the second moment of the distribution of the Lyapunov exponent of the w.f. on the samples of one-dimensional (1-D) disordered system of finite system size and with a long-range structural correlation [3]. The distribution converges toward delta-function as the system size increases and hence the w.f. of the system is exponentially localized with probability 1 in infinite system. However, we found a slowly convergent property of the distribution with increasing the system size by observing its variance, i.e., the second moment, which is in clear contrast with the normal convergent property in the case of normal disordered system with only short-range structural correlation. Encouraged by the above observation, we expect to find some new feature in the fluctuation of physical quantities of finite system. This
is our principal motif of studying the fluctuation of physical quantities of various disordered systems.

We study in this Letter the power spectrum and Allan variance of the system size dependence of the transmission amplitude $t_n$ of electron in 1-D disordered systems. A striking feature we find in the transmission coefficient $T(n)$ and the phase $\theta_t(n)$ of $t_n$ is the $1/f^\alpha$-type power spectrum and the corresponding Allan variance, which are universal for various disordered systems. Similar results have been reported by Nakamura et al. in the case of periodic system under electric field [4]. Their study was concentrated to the Stark-ladder case but we remark in this Letter that our results represent a universal law which governs more general 1-D quantum systems.

We study the following simple tightly-binding model to represent a variety of disordered systems. It is described as

$$c_{n+1} + \epsilon_n c_n + c_{n-1} = E c_n,$$

where $c_n$ denotes the amplitude of the electronic w.f. at site $n$, $\epsilon_n$ does the site energy at $n$, and $E$ the energy of the quantum state. The transfer energy has been determined as unity. The finite sample of system size $N$ locates over the sites from 1 to $N$ and it connects with two perfect conductors having constant $\epsilon_n (=0)$ for $n \leq 0$ and $n \geq N+1$ on both sides. When we adopt scattering condition in the electronic state by introducing the incoming plane wave $\exp(ikn)$ from the left ($n \leq 0$), we can obtain the reflected wave $r_N \exp(-ikn)$ ($n \leq 0$) to the left and the transmitted wave $t_N \exp(ikn)$ ($n \geq N+1$) to the right of the sample. The reflection coefficient $r_n = |r_n| \exp(i\theta_r(n))$ and transmission coefficient $t_n = |t_n| \exp(i\theta_t(n))$ are numerically calculated by the transfer matrix method.

The reflection coefficient $R(n) = |r_n|^2$ and the transmission coefficient $T(n) = |t_n|^2$ satisfy the current conservation law $R(n) + T(n) = 1$.

As the sample system, we firstly consider a purely random system in which $\{\epsilon_n, n = 1, \ldots, N\}$ are mutually independent random variables with the probability

$$P(\epsilon_n) = \begin{cases} 1/W, & |\epsilon_n| < 1/2W, \\ 0, & \text{otherwise}, \end{cases}$$

and call it Anderson model (A model). The structural correlation length of it is obviously vanishing. We secondary consider a disordered system with power law structural correlation in the sequence of the site energy $\{\epsilon_n\}$ by utilizing the modified Bernoulli map [5] as

$$X_{n+1} = \begin{cases} X_n + 2B^{-1}X_n^B, & (0 \leq X_n < 1/2), \\ X_n - 2B^{-1}(1 - X_n)^B, & (1/2 \leq X_n \leq 1), \end{cases}$$

$$\epsilon_n = \begin{cases} W, & (0 \leq X_n < 1/2), \\ -W, & (1/2 \leq X_n \leq 1), \end{cases}$$

where $B$ is a bifurcation parameter which controls the correlation of the sequence as follows: $(\epsilon_n \epsilon_{n+1}) \sim n^{-(2-B)/(B-1)}$. We call it modified Bernoulli model (B model). Due to the power-law structural correlation with diverging structural correlation length, the model is considered to be outside of the universality class of
the usual 1-D random system with finite correlation length. The model locates between the usual random system and periodic system with long-range order.

Numerical sample of \( T(n) \) of A model is exemplified in Fig. 1 for the case \( W = 0.15 \) and for the state \( E = 0 \). Because the transmission coefficient decays exponentially, we determine in Fig. 1(b) the mean straight line (mean exponential function) by the root-mean-square fitting and subtract the exponential part from Fig. 1(b). Then we can obtain the fluctuating part of \( \log T(n) \), i.e., \( \log T(n) \) fluct shown in Fig. 1(c). The distribution of the Lyapunov exponent \( \gamma \), determined for each sample by the slope of the line in Fig. 1(b), is shown in the hatch of Fig. 1(b). The average localization length \( \xi \) is defined by the sample-averaged Lyapunov exponent of \( T(n) \) as \( \xi = 2/(\gamma') \).

The power spectrum \( P(f) \) of the fluctuating part \( \log T(n) \) fluct defined as

\[
P(f) = \frac{1}{N^2} \sum_{j=0}^{N-1} \langle \log T(j+1) \rangle_{\text{fluc}} \exp(-2i\pi jfk),
\]

and the power spectrum of \( \theta_t(n) \) are shown in Fig. 2, where \( f_k = 0, 1/N, 2/N, \ldots, 1/2 \). It is surprising to find a universal power law in each of them irrespective of the kind of disorder whether the structural correlation length is finite or not. The corresponding Allan variance of \( T(n) \),

\[
\sigma^2(n) = \lim_{N \to \infty} \frac{1}{2N} \sum_{k=0}^{N-1} \left( M_n(k+1) - M_n(k) \right)^2,
\]
\[ M_n(k) = \frac{1}{n} \sum_{j=1}^{n} (\log T(nk + j))_{\text{fluc}}, \quad (7) \]

and one of \( \theta_t(n) \), which are shown in Fig. 3, again exhibit the corresponding interesting universal power law. Obviously, these two power laws exhibit non-stationary process. In addition it is remarkable to find that, even in our non-stationary process, the sum of the two indices of power of \( P(f) \) and \( \sigma^2(n) \) is equal to \(-1\), which is identical with the relation of the case of stationary process.

Until now, we have dealt only with \((\log T(n))_{\text{fluc}}\) to subtract the analytical exponential function which also produce the power spectrum of the same type. Moreover, we have the same result even when we calculate the power spectrum of \( T(n) \) itself. The later power spectrum also has enough physical meaning when the localization length \( \xi \) is bigger than the sample size \( N \). It is remarkable to mention that, in all of those cases, we have the universal power spectrum of the type \( 1/f^2 \) in \( T(n) \) and \( 1/f^{1.4} \) in \( \theta_t(n) \).

To summarize, we have studied the fluctuation of the transmission coefficient \( T(n) \) and of \( \theta_t(n) \) of the transmission amplitude in 1-D disordered systems, and have obtained some universal power law. We have obtained non-stationary power spectrum of \( 1/f^2 \) in the former and of \( 1/f^{1.4} \) in the later. Correspondingly, we also have obtained the Allan variance of \( n^1 \) in the former and of \( n^{0.4} \) in the later. Similar results have been obtained also for the reflection amplitude \( r_n \) [6].

Until now we have not got the reason why the fluctuation is so universal and how the indices of the power law of the power spectrum and Allan variance should be \(-2\) and \(1\) or \(-1.4\) and \(0.4\), respectively.

The further investigation is open for the future study.

References