Lyapunov Spectra of Correlated Random Matrices

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We have numerically studied Lyapunov spectra and maximal Lyapunov exponent in products of real symplectic correlated random matrices, each of which is produced by a modified Bernoulli map.

In dynamical systems with many degrees of freedom, the instability of tangent vector plays an important role when we consider statistical mechanics of the system. Lyapunov exponents, which are the exponential growth of the vectors on the trajectories, can be numerically estimated by products of real symplectic matrices.[1] On the other hand, it is well-known that $1/f$ type fluctuation is observed in the time-dependence of some physical quantity, such as total and one particle potential energy, in some dynamical systems.[2] However, the statistical property of the products of correlated pseudo-random matrices has not been fully studied, except for some cases.[3]

In this paper, we numerically study Lyapunov spectra of products of pseudo-random matrices with long-range correlation in the matrix sequence, each of which is produced by a modified Bernoulli map. (See below.) The statistical property of the sequence of the map, i.e., the correlation function shows power-low decay, has been well studied.[4] We can systematically investigate the influence of the correlation on the Lyapunov spectra.

We calculate products of $2N$-dimensional symplectic matrices $\{S^{(i)}\}$, $\boldsymbol{P}_n = \prod_i^n S^{(i)}$. First, we set a form of the symplectic matrices as follows:[1] $S^{(i)} = \begin{pmatrix} \boldsymbol{I} & \delta t\boldsymbol{I} \\ -\delta t\boldsymbol{H}^{(i)} & \boldsymbol{I} - \delta t^2\boldsymbol{H}^{(i)} \end{pmatrix}$, where $\boldsymbol{I}$ and $\boldsymbol{H}^{(i)}$ are N-dimensional identical matrix and real symmetric matrices, respectively. We adopt Hessian matrix based on one-dimensional dynamical system with nearest-neighbor interaction as the matrix configuration of the $\boldsymbol{H}^{(i)}$. As the result, the matrix form becomes tridiagonal with two corner elements caused by a periodic boundary condition and it has N independent random elements in a matrix $\boldsymbol{H}^{(i)}$ based on a conservation law , $\sum_{l=1}^N \boldsymbol{H}^{(i)}{}_{lk} = 0$. See Ref. 5) for the details. We used the N off-diagonal elements, $\{w_j^{(i)}(= \boldsymbol{H}_{j,j+1}^{(i)}), j = 1, \cdots, N-1\}$ and $w_N^{(i)} = \boldsymbol{H}_{N,1}^{(i)}$, as the independent variables.

Next, we generate the sequence of the matrices $\{w^{(i)}, i = 1, 2, \cdots\}$ by modified

Bernoulli map as follows:[4)]

$$X_{i+1} = \begin{cases} X_i + 2^{B-1}X_i^B & (0 \le X_i < 1/2) \\ X_i - 2^{B-1}(1-X_i)^B & (1/2 \le X_i \le 1) \end{cases},$$
$$w_j^{(i)} = -\epsilon X_i,$$

where B is a parameter which controls the correlation of the sequence, ϵ is strength of the $\boldsymbol{H}$. When we make the sequence, the different initial conditions of the map for each N independent elements of the matrix, $\{w_j^{(i)}, j = 1, \cdots, N\}$, are used. It is known that the correlation function of the symbolized sequence, $\{X_i\} \to \{Y_i\}$, decreases obeying power-law, $\langle Y_{n+1}Y_n\rangle \sim n^{-\frac{2-B}{B-1}}$, for large n. Then we numerically calculate the singular values of the matrix $\boldsymbol{P}_n$, $C_\infty = \lim_{n\to\infty}\left[\boldsymbol{P}_n^\dagger \boldsymbol{P}_n\right]^{\frac{1}{2n\delta t}}$, where the real symmetric symplectic matrix C_∞ is positive definite. Accordingly the Lyapunov exponents $\{\lambda_i\}$ are defined by $\lambda_i = \log \sigma_i(C_\infty)$, where $\sigma(i)$ denotes ith eigenvalue. We set the exponents in decreasing order and consider the positive upper half.

Figure 1(a) shows the scaled Lyapunov spectra $\lambda_i^*(=\lambda_i/\lambda_1)$ for some Bs. It follows that the functional form of the spectra $\lambda_i^*(i/N)$ drastically changes from straight line to curved one in non-stationary region ($B \ge 2$). Figure 1(b) shows the ϵ-dependence of maximal Lyapunov exponents (MLE) for some Bs. It is evident that the ϵ-dependence becomes mild as the correlation parameter B increases. Details for numerical simulation and the results will be published elsewhere.[6)]

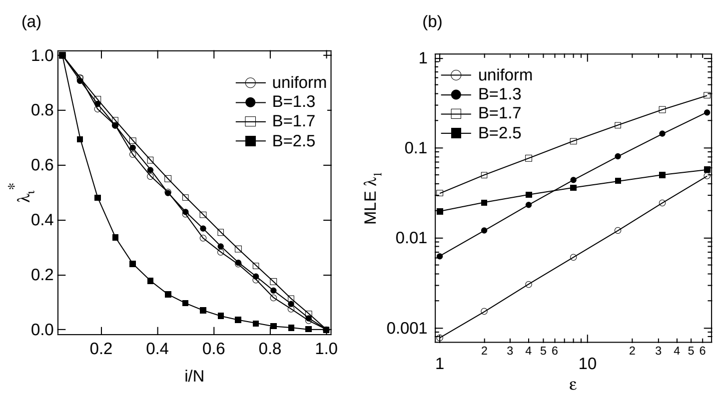

Fig. 1. (a) Scaled Lyapunov spectra as a function of i/N for some Bs. (b) ϵ-dependence of the MLE λ_1 for some Bs. We used N =16, δt =0.01. As a reference, a result computed by random number generator is shown by open circle.

References

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