Quantum Tunneling and Decoherence in Coherently Driven Double-Well Potential

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Contents

1. Introduction
2. Model
3. Numerical Results
4. Summary and Discussion
1 Introduction

We numerically investigate influence of a polychromatic perturbation on wave packet dynamics in one-dimensional double-well potential. Some interesting results are known in monochromatic perturbed double-well system, such as enhancement or suppression of tunneling between the wells, however tunneling dynamics in polychromatically perturbed double well does not seem to be clear. In order to make it clear, we consider tunneling dynamics the one-dimensional double-well driven by coherent external field. Especially we consider the following problem.

• Does coherent tunneling occur under the perturbation?
• Does one remain against strong perturbation?
• If the coherence of the tunneling dynamics is destroyed, what kind of motion becomes dominant?
2 Model

The model system is coherently driven one-dimensional double-well whose Hamiltonian is,

\[ H(p, q, t) = \frac{p^2}{2} + \frac{q^4}{4} - A(t) \frac{q^2}{2}, \quad (1) \]

\[ A(t) = a - \frac{1}{\sqrt{M}} \sum_{j=1}^{M} \epsilon_i \sin(\Omega_i t + \theta_i). \quad (2) \]

Here \( A(t) \) describes time-dependent perturbation and,

- \( q, p \): Canonically conjugate position and momentum.
- \( a \): The parameter determining the distance between the potential well.
- \( M \): The number of frequency component.
- \( \{\epsilon_i\} \): The \( i \)'s perturbation strength.
- \( \{\Omega_i\} \): Mutually incommensurate and order of unity frequencies of the external perturbation.
- \( \{\theta_i\} \): The initial phases of the external driving force.

To make some energy doublets and for simplicity we set \( a = 5, \epsilon_i = \epsilon = 0.1 \sim 1.0, \) and \( \theta_i \)'s are random numbers. We take off-resonant frequencies which are far from resonance in the corresponding unperturbed problem in order to avoid energy absorption due to resonance.
Time dependence of the potential, the initial wave packet, and some lowest unperturbed eigen energy levels are shown Fig. 1.

Fig. 1: The sketch of the potential at some moments, the initial packet and some eigen energy.

The classical phase space structure of this system are shown below.

Fig. 2: Points are plotted at $t = 2\pi n/\Omega_1$ in each cases.
3 Numerical Results

3.1 Tunneling

To calculate wave packet tunneling, we use the following Gaussian localized as the initial state, which is localized in the right well of the potential,

\[ \psi(q, t = 0) = \exp\left\{ -\frac{(q - q_0)^2}{2\sigma} \right\}. \]  \hspace{1cm} (3)

- \( q_0 \): the right bottom of the potential well.
- \( \sigma \): the initial spread of the packet.

We set \( \sigma = 0.3 \). This initial packet approximates equal-weight linear combination of the unperturbed lowest doublet. We then define \( P_L(t) \),

\[ P_L(t) \equiv \int_{-\infty}^{0} |\psi(q, t)|^2 dq \]  \hspace{1cm} (4)

which can be interpreted transition probability that the initially localized wave packet goes through the central energy barrier and reach the opposite well.
Fig. 3: Tunneling rate $P_L(t)$
We see coherent motion, irregular fluctuation and quasi-irregular motion as a intermediate motion.
3.1.1 Fourier Transform

To evaluate what frequency dominates the time-dependence of $P_L(t)$, we calculate and show the Fourier transform $I(\omega)$.

$$I(\omega) = \left| \int_0^T P_L(t) \exp(-i\omega t) dt \right|^2$$

where $T = 9.4 \times 10^3$.

![Graphs showing Fourier transform of tunneling rate $P_L(t)$](image)

Fig. 5: Fourier transform of tunneling rate $P_L(t)$

As $\epsilon$ and/or $M$ increase, some peaks appear and frequencies are distributed with finite-width around the peaks.
3.1.2 Quantitative description of perturbation dependence of tunneling rate and the corresponding classical dynamics

Fig. 6: Histograms of $P_L$ for some combinations $\epsilon$ and $M$.

From the figures the variance of the value of $P_L(t)$ can be a criterion which enables to describe the difference between the coherent and incoherent tunneling dynamics.
In order to estimate quantitatively the difference between coherent and incoherent motion, we use the variance of $P_L(t)$ as a degree of coherence

$$\Delta P_L \equiv \{\langle (P_L(t) - \langle P_L \rangle_T)^2 \rangle_T \}^{\frac{1}{2}}, \quad (6)$$

where $\langle \ldots \rangle$ denotes time average for the period $T$.

We show the degree of coherence and the maximum Lyapunov exponent of the corresponding classical system.

Fig. 7: Degree of coherence and Lyapunov exponents of the corresponding classical system.
The classification of the motion suggests the existence of the critical perturbation strength $\epsilon_c(M)$, over which the motion decoheres in quantum mechanics.
3.2 Wave Packet Dynamics

Next we consider the expectation value $\langle H(t) \rangle$ of the Hamiltonian and the deviation $\Delta H(t)$.

![Graph showing expectation value of the Hamiltonian for different values of $\epsilon$ and $M$.]  

*Fig. 9: Expectation value of the Hamiltonian*
Fig. 10: Expectation value of the Hamiltonian
Fig. 11: Standard deviation of the Hamiltonian
Fig. 12: Standard deviation of the Hamiltonian

When the perturbation strength $\epsilon$ exceeds $\epsilon_c(M)$, \textit{activation} becomes more dominant than tunneling and the motion changes incoherent one.
Next we show the plots ($\langle q \rangle \langle H \rangle$, $\langle q \rangle \langle p \rangle$) for some combination $\epsilon$ and $M$. 
Next we consider the uncertainty product $\Delta q \Delta p$ to evaluate how wave packet spreads in the phase space.

$$
\Delta q \Delta p \equiv \sqrt{\langle (q - \langle q \rangle)^2 \rangle} \sqrt{\langle (p - \langle p \rangle)^2 \rangle}.
$$

(7)

Fig. 13: Uncertainty product.

In the panel (a) and (b) coherent motion occur and the corresponding wave packets coherently tunnels between the wells. Coherent oscillations remain even for large perturbation strength. On the other In the incoherent motions (the panel (c) and (d) ), the wave packets spread over the phase space and never return to the initial ones within $T$. 
Finally we show contour plots of Hushimi representation \( \rho(x, p) \) for some typical cases.

![Contour plots of Hushimi representation](image)

**Fig. 14:** Hushimi representation.

In incoherent motion wave packet spreads over phase space with out symmetry.
4 4. Summary and Discussion

Summary

• Existence of some types of motion and critical value of perturbation strength.
• Perturbation can destroy the coherent tunneling for a perturbation strength above the critical value.
• Perturbation makes Recurrence time for incoherent motion much larger.

Discussion

• Long time behavior of quasi-irregular motion.
• What is the mechanism of change of wave packet traveling from tunneling to activation?
• Can it control somehow (e.g. OCT) ?
• What and how does classical chaos affect to the quantum dynamics (tunneling, coherence) ?
• Semiclassical and semiquantantal description of the incoherent tunneling
• Relation to stochastic perturbation which corresponds to the limit $M \to \infty$ and the situation without coherence.
• Resonance structure under polychromatic perturbation.