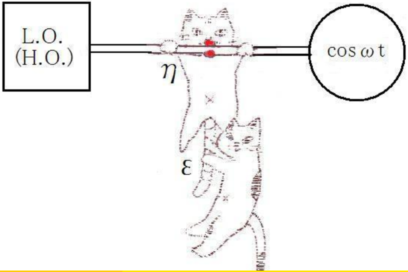
Irreversibility of Quantum States

Hiroaki Yamada

山田物理学研究所 (YPRL)

2019/xx/xx

- 1. Introduction
- 2. Time-reversal test
- 3. Irreversibility test
  (Electron-phonon system)
- 4. Life-time of time's arrow
  (Arold cats system)
- 5. Perspective



Introduction (Motivation) 1

•Motivation:

Is a macroscopic number of degrees of freedom (DOF) really necessary for destruction of the coherence and time-reversibility in the closed quantum system?

- Daily life world:

loss of initial memory due to chaos $\Rightarrow$ Irreversibility, ...

- Quantum world:

Non-chaotic behavior due to localization mechanism

Coupling with some appropriate degrees of freedom (DOF)

$\Rightarrow$ recovery of ability of losing memory, self-induced irreversibility,...

Introduction (Motivation) 2

Problem related to the irreversibility of closed quantum system:

•How does the localization change as perturbations from other DOF is introduced in 1DDS?

•What is difference between the 1DDS and periodic system as it are coupled with other degrees of freedom?

Delocalization, Irreversible energy transfer, Stationary, ??

Dynamics in 1DDS

- Tightly Binding Model of 1DDS

$$
\begin{aligned}
H(t) &= \sum_{n=1}^{N} |n\rangle V(n)(1+f(t))\langle n|\delta_t - J\sum_{n}(|n\rangle\langle n+1| + |n+1\rangle\langle n|), \\
\delta_t &= \sum_{m}\delta(t-\tau m), f(t) = \frac{\epsilon}{\sqrt{M}}\sum_{j=1}^{M}\cos(\omega_j t+\theta_j).
\end{aligned}
$$

  - $V(n) = Wv_n$: spatial disorder (disorder strength W,$v_n \in [-1,1]$)
  - ϵ: perturbation strength($\epsilon < 1$)
  - ω_i: incommensurate frequencies (M: number of colors)

$$
i\hbar\frac{\partial\phi(n,t)}{\partial t} = H(t)\phi(n,t),
$$

- usual Anderson model for $\tau \to 0$

Dynamics in QSM

•Quantum standard map

$$\begin{aligned} H &= \frac{\hat{p}^2}{2} + K\cos\hat{q}\delta_t, \\ \hat{U} &= \mathrm{e}^{-i\hat{p}^2 2/2\hbar}\mathrm{e}^{-iK\cos\hat{q}/\hbar}\mathrm{e}^{-i\hat{p}^2 2/2\hbar} \end{aligned}$$

−diffusion in momentum space $-\infty < p < \infty$

classical dynamics: normal diffusion

$$\langle (p-\langle p\rangle)^2\rangle = D_{cl}t, \quad D_{cl} \sim \frac{K^2}{2} \ \ \mathrm{for} K >> 1$$

quantum dynamics: localization

$$\langle (\hat{p}-\langle \hat{p}\rangle)^2\rangle = \xi^2, \quad \xi \simeq \frac{D_{cl}}{2\hbar^2}$$

$\Rightarrow K \to K(1+f(t))$ with

$$f(t) = \frac{\epsilon}{\sqrt{M}}\sum_{j=1}^{M}\cos(\omega_j t + \theta_j).$$

Dynamics in 1DDS (probability distribution)

Ansemble-averaged probabilistic function and MSD:
$P(n,t) = \langle |\Psi(n,t)|^2 \rangle, \;\; m_2 = \langle (\Delta n)^2 \rangle$

• Localization:

$$P(x,t) \sim \exp\{-|x|/\xi\}, \;\; m_2(t) \sim \xi^2$$

• Delocalization(Normal diffusion):

$$P(x,t) \sim \exp\{-x^2/2Dt\}, \;\; m_2(t) \sim Dt^2$$

Increase of the strength ϵ and/or L:
Exponential Localization $\Rightarrow$ Gaussian function (a solution in diffusion equation in the stochastic process

"classicalization " of the quantum wavepacket

Time-reversality 1

Time-reversal test:

- First, we evolve an initial quantum state $|\Psi_0>$ forward in time by applying the unitary operator U^T at the reversal time T
- Next, we perturb the evolved state by P, and evolve the perturbed state backward in time by applying the time-reversed unitary operator U^{-T}.

Time-reversal characteristics:

At $t = 2T$, the difference between the perturbed time-reversed state $|\Psi'_0> = U^{-T} P U^T |\Psi_0>$ and the initial state $|\Psi_0>$

$$\boxed{\delta Q = \frac{[< \Psi'_0|\hat{Q}|\Psi'_0> - <\Psi_0|\hat{Q}|\Psi_0>]}{<\Psi_0|\hat{Q}|\Psi_0>},}$$

where $\hat{Q}$ is an appropriate observable

Time-reversality 2 (time-reversal characteristics)

Time-reversal characteristics measured by the MSD $M(t)$

$$\boxed{\mathcal{R}(\eta) = \frac{|M_\eta(2T) - M_0(2T)|}{M_0(T)}.}$$

$\eta-$shift operator for the perturbation $\hat{P}(\eta)$ applied at T,

•"perpendicular shift"

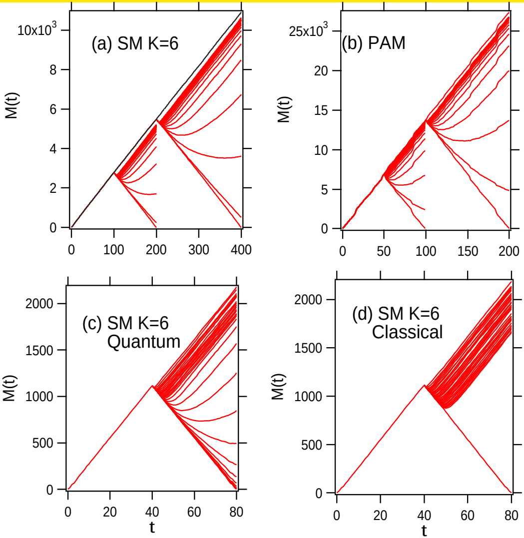
$$\hat{P}_y(\eta) = \exp\{i\eta\hat{x}/\hbar\} = exp\{\eta\partial/\partial y\}.$$

• "parallel shift"(y canonically-conjugate to the diffusion space x)

$$\hat{P}_x(\eta) = exp\{i\eta\hat{y}/\hbar\}$$

$y = q$, $x = p$ for SM, and $y = p$, $x = q$ for PAM and HM

Time-reversality 3



Time-reversal test (perpendicular shift)

Time-reversality 4

In classical dynamics:

$$\begin{aligned}\theta_{2T} &= \theta_0 + c_1\eta + T(\omega(I_0) - \omega(I_0 + c_2\eta)),\\ I_{2T} &= I_0 + c_2\eta.\end{aligned}$$

•Integrable case:

$$|p_{2T} - p_0| \sim \left|\frac{\partial P(I_0,\theta_0)}{\partial\theta}\frac{\partial\omega'(I_0)}{\partial I}c_2\eta T\right|$$

$$\mathcal{R}_{cl} = \frac{|M_\eta(2T) - M_0(2T)|}{M_0(T)} \sim \eta T.$$

control the accuracy:

$$\boxed{\eta \sim 1/T.}$$

Time-reversality 5

•Chaotic motion:

$$\mathcal{R}_{cl} \sim 2 - \frac{\tau_d(\eta)}{T}, \quad \tau_d(\eta) = \frac{\log(C/\eta)}{\lambda}$$

control the accuracy:

$$\boxed{\eta \sim C\mathrm{e}^{-\lambda T},}$$

•threshold value of the perturbation strength η_{th}
$\Rightarrow$ Least Quantum Perturbation Unit (LQPU) :

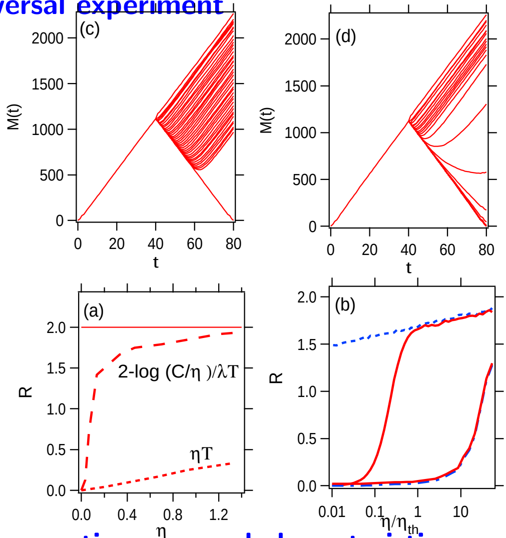
$$\eta_{th} = \frac{2\pi\hbar}{\Delta x(T)}.$$

Then the sweep area $A = 2\pi \times \eta$,

$$\eta_{th} = \hbar.$$

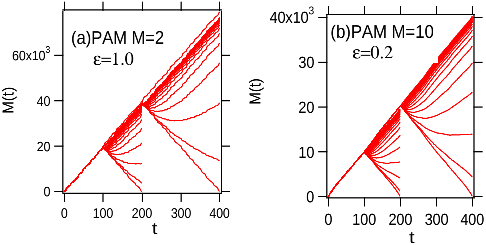
Time-reversality 6

- SM: **time-reversal experiment**

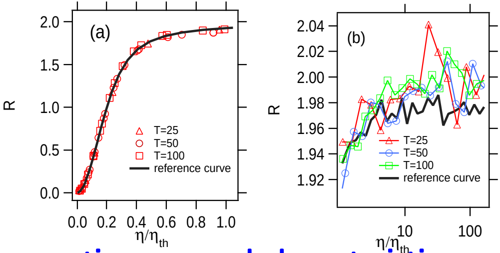


time-reversal characteristics

- AM:

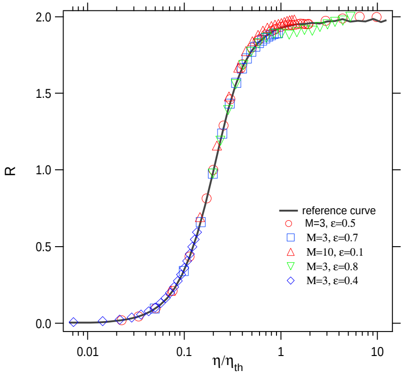


time-reversal experiment (perpendicular shift)

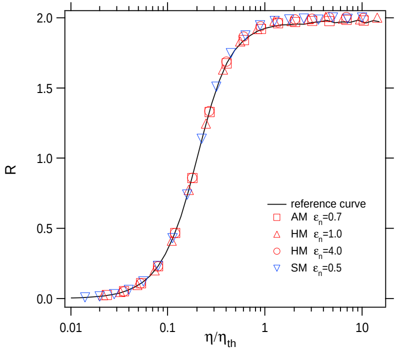


time-reversal characteristic

Universal curve as a function of the scaled perturbation strength $\frac{\eta}{\eta_{th}(T)}$,



time-reversal characteristic (PAM)



time-reversal characteristic (stochastic model)

Time-reversality 9

In the limit as $T \to \infty$,

$$\mathcal{R} = F(\frac{\eta}{\eta_{th}(T)}),$$

the difference $\Delta M_\eta \equiv M_\eta(T, T+\tau) - M_0(T, T+\tau)$

$$\Delta M_\eta = G(\eta, \tau) = D\tau F\left(\frac{\eta}{\eta_{th}(\tau)}\right),$$

Accordingly,

$$\begin{aligned} & \frac{M_\eta(T, T+\tau) - M_0(T, T+\tau)}{M_0(T, T)} \\ = \quad & \frac{\tau}{T} F\left(\frac{\eta}{\eta_{th}(T)} \frac{\eta_{th}(T)}{\eta_{th}(\tau)}\right) = \frac{\tau}{T} F\left(\frac{\eta}{\eta_{th}(T)} \left\{\frac{\tau}{T}\right\}^{\chi}\right), \end{aligned}$$

where $\eta_{th}(T) \propto T^{-\chi}$. $\chi = 1/2$ for a perpendicular shift and $\chi = 0$ for a parallel shift.

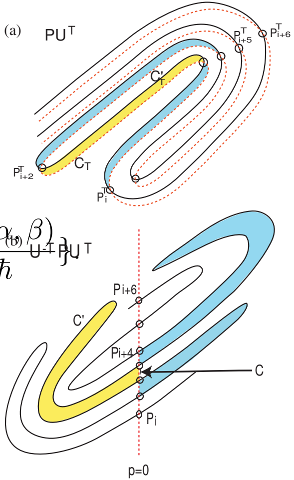
Time-reversality 10

•Semicassical interpretation of LQPU:

Initial manifold:
$\mathcal{A}_\alpha = \{(q,p) \mid q = \alpha \}$:
The semiclassical wavefunction
at $q = \beta$

$$\Psi(\alpha,\beta) = \sum_\ell [\frac{\partial S^\ell(\alpha,\beta)}{\partial\alpha\partial\beta}]^{-1/2} \exp\{i\frac{S^\ell(\alpha,\beta)}{\hbar}\}.$$

Observation manifold:
$\mathcal{B}_\beta = \{(q,p) \mid q = \beta\}$
the time-reversed Lagrangian
manifold
$U_{\mathrm{cls}}^{-T} \cdot P_{\mathrm{cls},\eta} \cdot U_{\mathrm{cls}}^{T}(\mathcal{A}_\alpha)$.



generation of the intersection

Time-reversality 11

$$p = -\frac{\partial S(\alpha, \beta)}{\partial \beta}.$$

The contor:

$$\begin{aligned} \Delta\psi(P_\ell, P_{\ell'}) &= \oint_{C+C'} pdq/\hbar = \oint_{C_T+C'_T} pdq \\ &\sim (q_\ell^T - q_{\ell'}^T)\eta = \Delta q_T \eta \end{aligned}$$

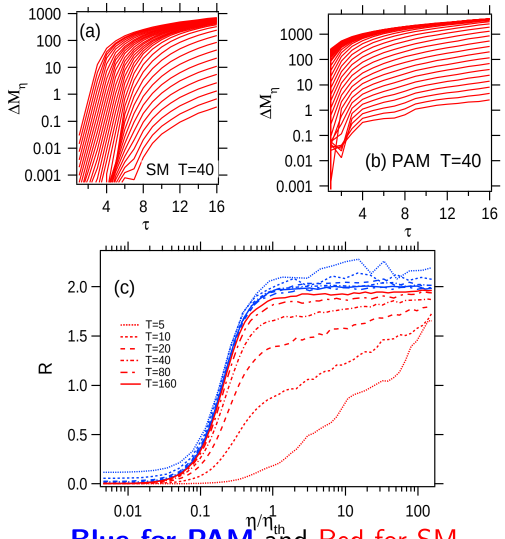
The quantum interference condition of the spread of the wavefunction Δq_T

$$\Delta q_T \eta/\hbar \ll 1.$$

LQPU:

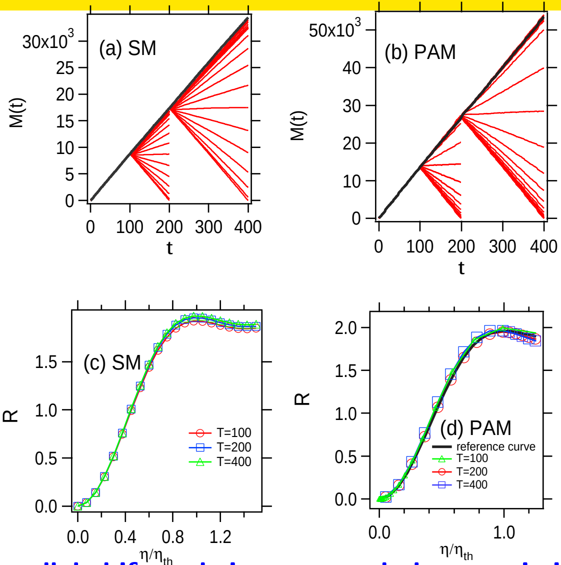
$$\boxed{\eta_{th} = \hbar/\Delta q_T}$$

$$\Delta M_\eta(T, T+\tau) \equiv M_\eta(T, T+\tau) - M_0(T, T+\tau)$$



Blue for PAM and Red for SM

Time-reversality 13



parallel shift and time-reversal characteristics

Time-continious models

•Haken-Strobl model (HSM)

$$H_{HSM}(\hat{p},\hat{n}) = 2\cos(\hat{p}/\hbar) + \sum_n \epsilon_s N(n,t)|n><n|,$$

The $N(n,t)$ is spatio- and temporal-uncorrelated Gaussian noise at site n

$$<N(n,t)>=0, <N(n,t)N(n',t')>=\delta_{nn'}\delta(t-t'),$$

•Stochastic pendulum model (SPM)

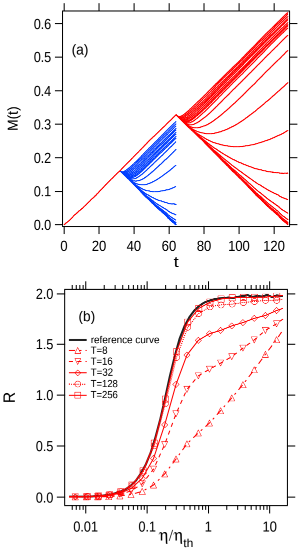
$$H_{SPM}(\hat{p},\hat{q}) \;\; = \;\; \frac{\hat{p}^2}{2} + (K+\epsilon_s N(t))\cos\hat{q},$$

The $N(t)$ represents a Gaussian stochastic noise satisfying

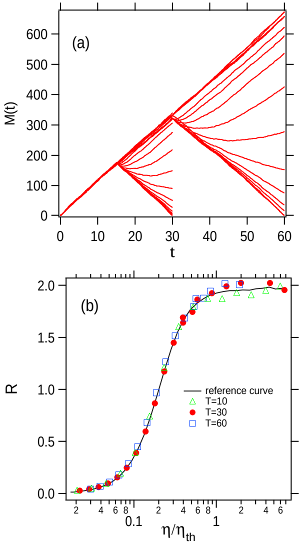
$$<N(t)>=0, \;\; <N(t)N(t')>=\delta_{tt'}.$$

In the classical limit $\hbar \to 0$, it has a classical counterpart whose trajectory

Time-reversality 15



perpendicular shift of SPM



perpendicular shift of HSM

Time-reversality 16 (Summary)

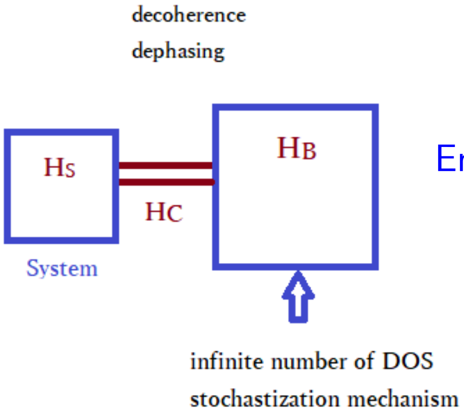
Result 1: In the time-reversal characteristic of quantum normal diffusion, there always exists a universal characteristic strength of P called the least quantum perturbation unit (LQPU), which is proportional to the Planck constant.

Result 2: In deterministic quantum maps, the time-reversal characteristic of quantum normal diffusion converges to a universal curve independent of the details of the system in the large limit of the reversal time.

Result 3: The universal features observed also hold exactly for the normal diffusion process of stochastically perturbed quantum maps.

Usual electron-Phonon system

•Heat-bath model:



Energy flow between H_S and H_B

Irreversibility test of Electron-Phonon system

- Electron with oscillatory perturbation (Nonautonomous system)

$$\begin{aligned} H_I &= H_{el} + H_{osc,L}(\{\Omega_j t\}), \\ H_{osc,L}(\{\Omega_j t\}) &= \sum_{n=1}^{N} \sum_{j=1}^{L} \epsilon_j \cos \Omega_j t |n\rangle V(n) \langle n| \end{aligned}$$

- Electron-linear oscillator system (Autonomous system) $(J_j = -i\hbar \frac{\partial}{\partial \phi_j})$

$$H_I^{aut} = H_{el} + H_{osc,L}(\{\phi_j\}) + \sum_{j=1}^{L} \Omega_j J_j,$$

$$\boxed{\mathrm{e}^{-iH_I^{aut} t/\hbar} = \exp\left\{ -i \sum_{j=1}^{L} \Omega_j J_j t/\hbar \right\} \hat{T}\exp\left\{ -i \int_0^t ds H_I(\{\Omega_j s + \phi_j\})/\hbar \right\}}$$

Irreversibility test 2 (Electron-phonon system)

•Electron-Harmonic oscillator system (Autonomous system)

$$
\begin{aligned}
H_I &= H_{el} + H_{ph,M} + H_{int,M}, \\
H_{ph,M} &= \sum_{j=1}^{M} \left(\frac{p_j^2}{2} + \frac{\omega_j^2 q_j^2}{2} \right), H_{int,M} = \sum_{n=1}^{N} \sum_{j=1}^{M} b_j q_j |n\rangle V(n) \langle n|, \\
H_{int,M} &= \sum_{n=1}^{N} \sum_{j=1}^{M} \frac{b_j}{\omega_j} |n\rangle V(n) \langle n| (a_j^\dagger + a_j) \sqrt{\frac{\hbar\omega_j}{2}}
\end{aligned}
$$

matrix elements: $n_j, n_j^{'} << N_j^*$,

$$
\begin{aligned}
& \langle N_1^* + n_1, ..., N_M^* + n_M | H_{int,M} | N_1^* + n_1^{'}, ..., N_M^* + n_M^{'} \rangle \\
\simeq & \sum_{n=1}^{N} \sum_{j=1}^{M} b_j N_j^* |n\rangle V(n) \langle n| \delta_{n_j, n_j^{'} \pm 1} \sqrt{\frac{\hbar\omega_j}{2}}
\end{aligned}
$$

Irreversibility test 3 (Electron-phonon system)

$$J_j|m_j\rangle = m_j\hbar|m_j\rangle, \langle\{\phi_j\}|\{m_j\}\rangle = \prod_{j=1}^{M}\frac{e^{im_j\phi_j}}{\sqrt{2\pi}},$$

matrix elements:

$$\langle\{\phi_j\}|H_{osc,L}(\{\phi_j\})|\{\phi_j^{'}\}\rangle = \sum_{n=1}^{N}\sum_{j=1}^{M}V(n)\epsilon_j\delta_{n_j,n_j^{'}\pm 1}/2$$

•Correspondence

$$\begin{aligned}
\Omega_j &\leftrightarrow \omega_j \\
|\{N_j^* + n_j\}\rangle &\leftrightarrow |\{m_j\}\rangle \\
\epsilon_j &\leftrightarrow \sqrt{\frac{2b_j^2 N_j^{*2}\hbar}{\omega_j}}
\end{aligned}$$

Irreversibility test 6 (Electron-phonon system)

- **Initial quantum state**

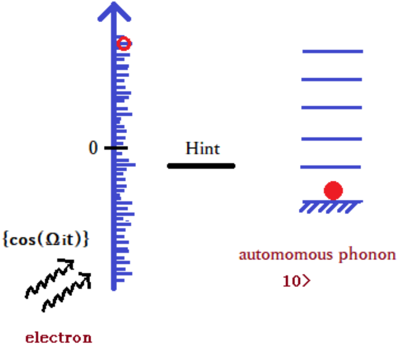
Disordered electron+Linear oscillator+Autonomous phonon+ Interaction H_{int} (b :coupling strength)

Electron: an excited state

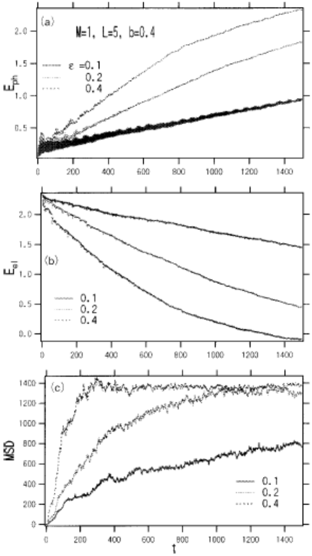
Autonomous phonon: ground state

Monitor: Energies and MSD

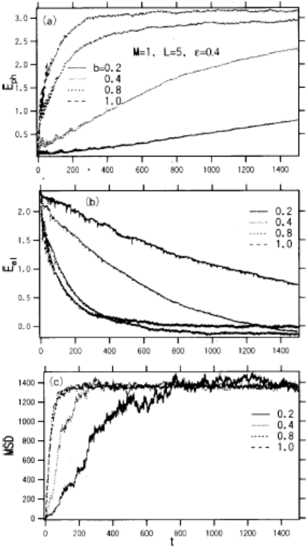
$$
\begin{aligned}
E_{ph}(t) &= \langle \Psi(t)|H_{ph}|\Psi(t)\rangle \\
E_{el}(t) &= \langle \Psi(t)|H_{el}|\Psi(t)\rangle \\
m_2(t) &= \langle \Psi(t)|(n-\langle n\rangle)^2|\Psi(t)\rangle
\end{aligned}
$$



Irreversibility test 7 (Electron-phonon system)



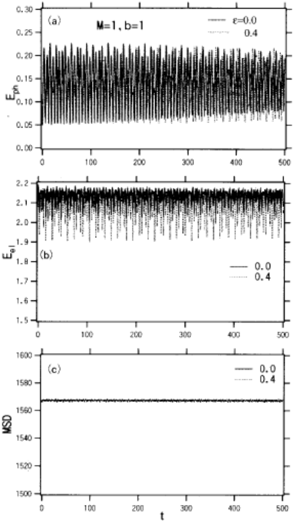
$M = 1, L = 5, b = 0.4$



$M = 1, L = 5, \epsilon = 0.4$

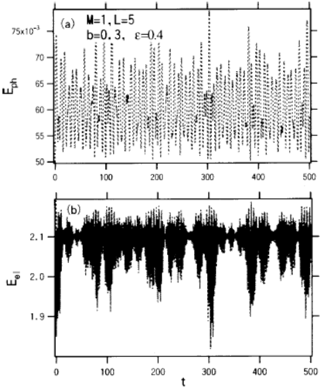
Irreversibility test 8 (Electron-phonon system)

• Disordered lattice ⇒ Periodic lattice



$b = 1.0$, $\epsilon = 0, 0.4$

Binary periodic lattice



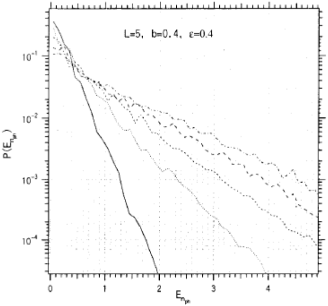
$b = 0.3$, $\epsilon = 0.4$

Recursive!

Time-dependent matrix elements of autonomous phonon system

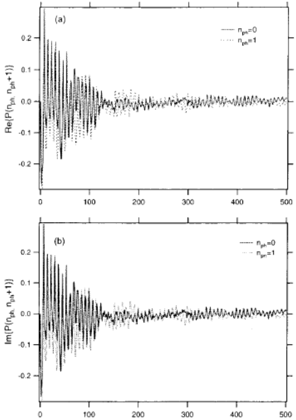
$$\text{Diagonal term}: P(n_{ph}) = \langle n_{ph}|\Psi\rangle\langle\Psi|n_{ph}\rangle$$

$$\text{Off}-\text{diagonal term}: P(n_{ph}, n_{ph}+1) = \langle n_{ph}|\Psi\rangle\langle\Psi|n_{ph}+1\rangle$$



$t = 100, 200, ..., 500$

Boltzmann-like distribution

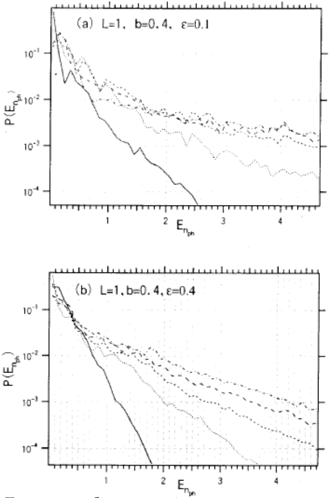


Non-interference

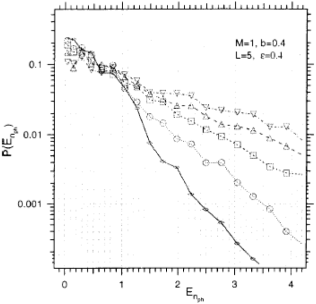
Irreversibility test 10 (Electron-phonon system)

Harmonic phonon $\Rightarrow$ Anharmonic phonon (quartic oscillator)

Energy flow

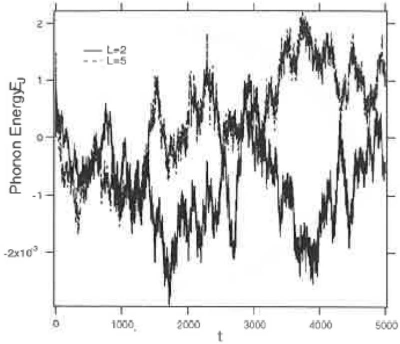


$L = 1$, $b = 0.4$, $\epsilon = 0.1, 0.4$

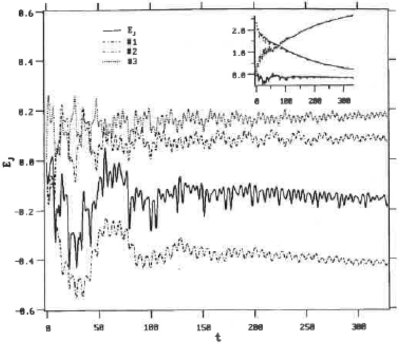


$P(n_ph)$**:neraly Boltzman-like distribution**

- Phonon energy: $\langle E_J(t) \rangle$



Energy of the linear oscillators



Energy of the linear oscillators

Irreversibility test 12 (Summary)

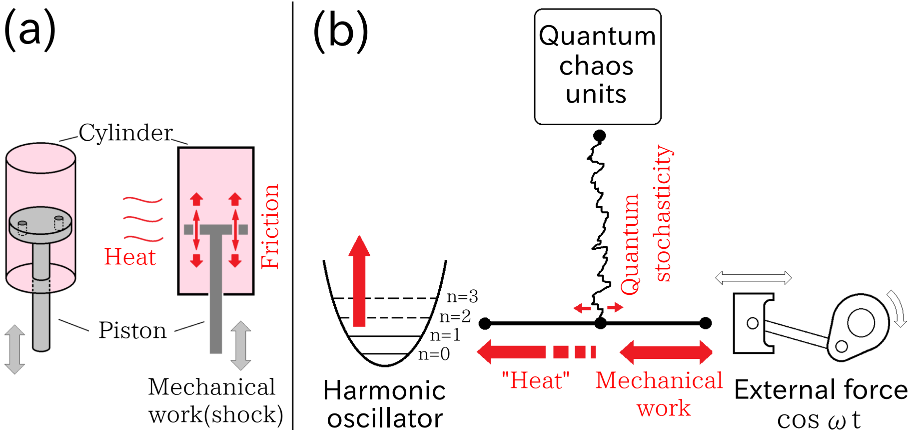
1D disordered electron+Linear oscilation+Autonomous phonon+ Interaction H_{int}:

The the total nuber of DOF of the total system is three $(L = 1, M = 1)$.

Results

- Irreversible energy fow from the excited electronic state to phononic state
- Boltzman-like distribution and dephasing are observed in the harmonic phonon state
- In the periodic systems, such a irreversible energy flow does not occur, it becomes recursive

Macroscopic damper and quantum damper



Life-time of time's arrow 2 (coupled cat maps)

$$\hat{H}_{tot} = \hat{H}_{HO} + \hat{H}_{KR} + \eta \hat{q} \cos \hat{I}_1 \cos \omega t,$$

$$\begin{aligned}
\hat{H}_{HO} &= \frac{\hat{p}^2}{2} + \Omega^2 \frac{\hat{q}^2}{2} \\
\hat{H}_{KR} &= \frac{\hat{I}_1^2 + \hat{I}_2^2}{2T} + \hat{V}_{KR} \sum_{m=-\infty}^{\infty} \delta(t - mT) \\
\hat{V}_{KR} &= -K \sum_{i=1}^{2} K \frac{\hat{\theta}_2^2 + \hat{\theta}_2^2}{2} + \varepsilon \cos(\hat{\theta}_1 - \hat{\theta}_2).
\end{aligned}$$

boundedness of phase space: $(I_i, \theta_i) \in [0, 2\pi] \times [0, 2\pi]$, and the Hilbert space $N_1 = N_2$, $\hbar = 2\pi / N_i \Leftrightarrow$ Standard map: unbounded phase space

Life-time of time's arrow 3 (coupled cat maps)

- Energy transfer process

$$\hat{b}_\tau - \hat{b} \simeq -\frac{\eta(e^{i\nu T} - 1)}{2\nu\sqrt{2\hbar\Omega}} \sum_{k=0}^{\tau-1} e^{ik\nu T} \hat{f}_k,$$

$$E_\tau - E_0 = \sum_{s \le \tau-1} A_s, \quad A_s = \mu \sum_{k=-s}^{s} C_{r_s}(k) \mathrm{e}^{-i\nu kT}, \mu \equiv \eta^2 \frac{|\exp^{i\nu T} - 1|^2}{8\nu^2}$$

- Autocorrelation function

$$C_{r_s}(k) = \begin{cases} \left\langle \hat{f}_s \hat{f}_{s-k} \right\rangle & \text{for } k \ge 0, \\ C_{r_s}(k) = C_{r_s}(-|k|)^* & \text{for } k < 0. \end{cases}$$

$$C_{r_s}(k) = \left\langle \cos^2 I \right\rangle \delta_{k,0} = \frac{1}{2}\delta_{k,0} \Rightarrow \langle E_\tau \rangle \sim \mu \left\langle f(\hat{I})^2 \right\rangle \tau$$

Life-time of time's arrow 4 (coupled cat maps)

- complex variables (a, a^*) obey the Gaussian distribution

$$P(a^*, a) \propto \mathrm{e}^{-|a|^2/\langle |a(t)|^2 \rangle} \Rightarrow P(E) \propto e^{-E/\langle E \rangle_\tau}$$

parameters	value
K	10
T	10^2
Ω	1
ω	$1+\sqrt{2}/T$
ξ	$4.25\hbar^2$
θ_1'	$\sqrt{2}$
θ_2'	$-\sqrt{5}$
η	5×10^{-4}
$\hbar$	$2\pi/N$
N	$2^4 \sim 2^8$
μ	$\sim 2.64 \times 10^{-4}$

Life-time of time's arrow 5 (coupled cat maps)

•Fourier transform of the quantum autocorrelation function

$$F_\tau(z) \equiv \sum_{k=-\tau}^{\tau} C_{r_\tau(k)} \mathrm{e}^{-izTk},$$

for $z = \nu = (\Omega - \omega)$ the classical chaotic decay of autocorrelation function if τ is less than the "lifetime" τ_L:

$$\tau_L \sim \begin{cases} CN_{\mathrm{dim}}^2 = CN^4 & \text{for} \quad z \neq 0 \\ CN_{\mathrm{dim}} = CN^2 & \text{for} \quad z = 0, \end{cases}$$

$$C(z) \sim Cr_{cls}(0) / \sum_{s=-\infty}^{\infty} Cr_{cls}(s) \mathrm{e}^{-izTs}$$

with the classical correlation function $Cr_{cls}(s)$.

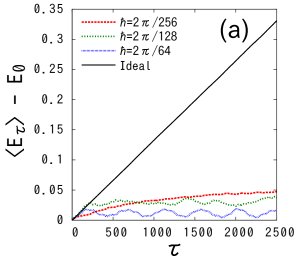
Life-time of time's arrow 6 (coupled cat maps)

- Absorbed energy and energy distribution

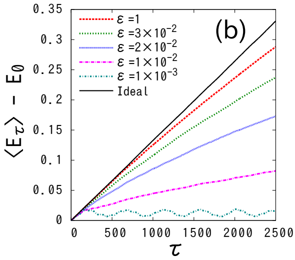
$$\langle E\rangle_\tau - E_0 \propto \tilde{A}\tau ??$$

$\Rightarrow$ stationariness ?, Boltzmann distribution?

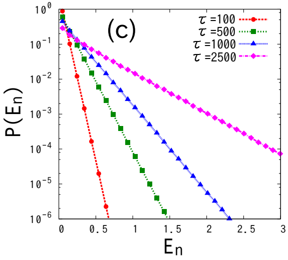
single cat



twin cats



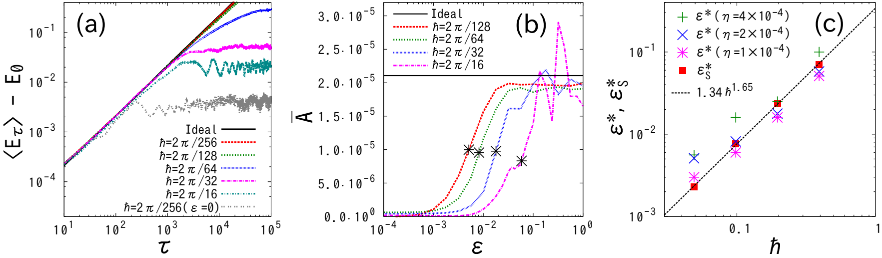
twin cats



Life-time of time's arrow 7 (coupled cat maps)

- $\hbar-$dependence of the energy absrbtion

Absrbtion rate $\tilde{A}$ and the thereshold value ε^*



$$\Rightarrow \varepsilon^* \propto \hbar^{1.65}$$

Coupled kicked rotors (CKR) with linear oscillator $\hat{J}$

$$
\begin{aligned}
H(\hat{\mathbf{I}}, \hat{\theta}, t) &= (\hat{I}_1^2 + \hat{I}_2^2)/2 + \delta_T(t)[V(\hat{\theta}_1) + V(\hat{\theta}_2) + \varepsilon V_{12}(\hat{\theta}_1, \hat{\theta}_2)] \\
V_{12}(\hat{\theta}_1, \hat{\theta}_2) &= \cos(\hat{\theta}_1 - \hat{\theta}_2), \quad \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)
\end{aligned}
$$

$(\hat{\theta}_i, I_i) \in [0, 2\pi] \times [0, 2\pi]$, and the Hilbert space $N_1 = N_2$, $\hbar = 2\pi/N_i$.

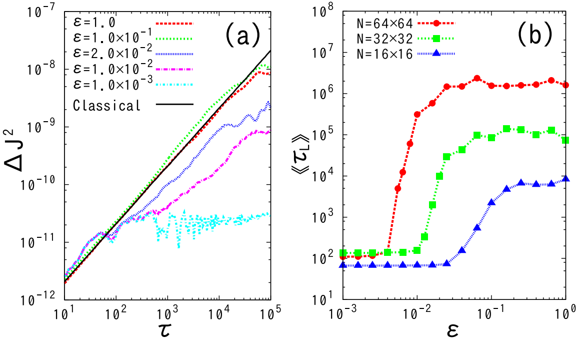
$$
\hat{H}_{tot}(t) = \hat{H}(\hat{\mathbf{I}}, \hat{\theta}, t) + \eta \sin(\hat{I}_1) \cos\hat{\phi} + \omega \hat{J}, \langle \phi | J \rangle \propto \mathrm{e}^{-iJ\phi/\hbar}
$$

Initial state: $|\Psi_0\rangle = |\psi_0\rangle \otimes |J = 0\rangle$, $|\psi_0\rangle = |I_1 = N/2\rangle \otimes |I_2 = N/2\rangle$

Spread of action and averaged Lifetime

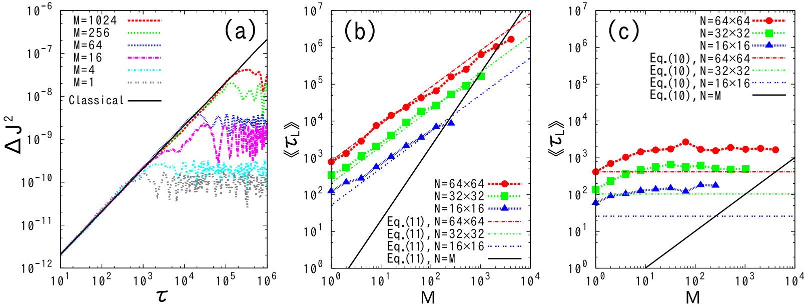
- $\varepsilon-$dependence

$$\Delta J^2(\tau) = \langle (\hat{J} - \langle \hat{J} \rangle)^2 \rangle$$



Numerically estimated averaged Lifetime $\langle\langle \tau_L \rangle\rangle$
by $|\alpha(\tau) - 1| > 1/2$, $\Delta J^2 \propto \tau^{\alpha}$

- $\hbar-$dependence

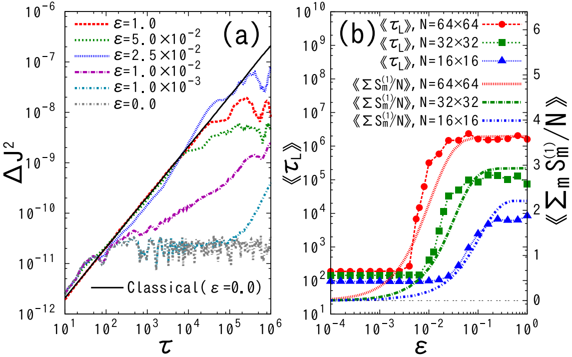


$N_1 = N_2 = 32$ **for (b)**$\omega = \sqrt{2} \neq 0$**, (c)**$\omega = 0$

Life-time of time's arrow 11 (coupled cat maps)

Coupled kicked rotors (CKR) $H(\hat{\mathbf{I}}, \hat{\theta}, t)$ without any oscillator

- Spread of action and averaged Lifetime of irreversibility
- $\varepsilon-$dependence

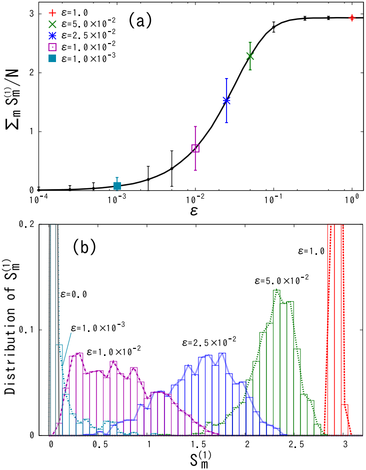


Life-time of time's arrow 12 (coupled cat maps)

- Entanglement entropy

$$\hat{U}_{CKR}|m\rangle = \mathrm{e}^{-i\gamma_m}|m\rangle,$$
$$m = 1, 2, ..., N_{tot}(= N_1 N_2)$$

$$S = \frac{1}{N_{tot}} \sum_{m=1}^{N_{tot}} S_m$$
$$S_m = -Tr\hat{\rho}^{(1)} \log \hat{\rho}^{(1)},$$
$$\hat{\rho}^{(1)} = Tr_2|m\rangle\langle m|$$



Entropy distribution

Life-time of time's arrow 13 (summary and problem)

Result

- We proposed a very simple model of a quantum damper as an application of classically decay-able correlation in coupled quantum kicked rotors.
- This model consist of only three quantum degrees of freedom and evolves by a time-reversal unitary dynamics.
- It can realize an almost ideal irreversible transformation of the mechanical work into the energy of the reservoir characterized by a Boltzmann-like energy distribution with an effective temperature.

A complete understanding of the relation between the duration time of stationary flow and the entanglement among the chaotic system is still an open problem.

Remaining problems

What are the quantum states that can produce irreversibility and dissipation?

- At the very least, it is related to the spread of the wave packet doing normal diffusive movement.
- In that sense, the quantum interference effect that causes localization phenomena is also destructed, and this is also related to the phenomenon that delocalization of Anderson localized state and Anderson transition occur.

References-1

Irreversibility of Electron-phonon system

•[HY96] H. Yamada and K. S. Ikeda, Spontaneously induced irreversible energy transport in closed electron-phonon, Phys. Lett. A **222**, 76 (1996).

•[HY02] H. Yamada and K. S. Ikeda, Anderson Localized State as a Pre-Dissipative State: Irreversible Emission of Thermalized Quanta from Dynamically Delocalized State, Phys. Rev. E 65, 046211-1-17(2002)

•[HY03] H. Yamada, Delocalization and Dissipative Property in 1D Disordered System with Oscillatory Perturbatuin, Proc. of Japan-Italy Joint Workshop on Quantum Open System, Quantum Chaos and Quantum Measurement: Fundamental Aspects of Quantum Physics, Ed. by L. Accardi and S. Tasaki, 80-99 (World Scientific 2003)

References-2

Time-Reversality test

•[HY10] H.S.Yamada and K.S.Ikeda, Universal Irreversibility of Normal Quantum Diffusion, Phys. Rev. E **82**, 060102(R)(2010).

•[HY12a] H.S. Yamada and K.S. Ikeda, Time-reversal Characteristics of Quantum Normal Diffusion, Eur. Phys. J. B 85, 41(2012).

•[HY12b] Time-reversal Characteristics of Quantum Normal Diffusion: Time-continuous Models, Eur. Phys. J. B 85,195 (2012).

,

References-3

Life-time of Time's arrow in coupled cats map system

•[HY16a] F. Matsui, H.S. Yamada and K.S. Ikeda, Correlation between irreversibility and entanglement in classically chaotic quantum kicked rotors, Europhys. Lett. 114, 60010(2016)

•[HY16b] F. Matsui, H.S. Yamada and K.S. Ikeda, Measuring lifetime of correspondence with classical decay of correlation in quantum chaos, Europhys. Lett. 113, 4008(2016)

•[HY18] F. Matsui, H.S. Yamada and K.S. Ikeda, A Quantum Damper, Physica A **523**, 631-641(2019) ; arXiv.1709.03115v2

Review of Fidelity

•[Fidelity1] F. Mintert, A. R. R. Carvalho, M. Kus, and A. Buchleitner, Phys. Rep. **415**, 207(2005).

•[Fidelity2] F. Haug, M. Bienert, and W. P. Schleich, T. H. Seligman and M. G. Raizen, Phys. Rev. A **71**, 043803 (2005).

•[Fidelity3] T. Gorin, T. Prosen, T. H. Seligman and M. Znidaric, Phys. Rep. **435**, 33-156 (2006).

•[Fidelity4] Ph. Jacquod and C. Petitjean, Adv. Phys. **58**, 67 (2009).