<メモ>

① Langevin eq ⟹ F-P eq.

確率過程の取扱い.

◎ Kubo の理論.

◎ Novikov の定理 etc.

つかう

◎ Invariant-imbedding app.

② Tallis 統計的取扱いを うまく手で作ることで.

べき相関 Random 系 との関係は 進められるかもしれない

① Shmidt の関数eq の解法で Clarke Model をやる.

Cauchy 分布

① $\boxed{\text{点スペクトル}}$ $\boxed{\text{連続スペクトル}}$

< Langevin eq. と Fokker-Planck equation >

Langevin eq.

$$\boxed{\frac{dx}{dt} = \underbrace{\alpha(x,t)}_{\text{決定論的}} + \underbrace{\beta(x,t)\eta(t)}_{\text{確率的}}}$$ ← Stratonovich型 とする

$\eta(t)$が ガウシアンノイズ 性質 ⇒

$$\begin{cases} \langle \eta(t_1)\eta(t_2) \rangle \equiv \delta(t_1 - t_2) \\ \left\langle e^{\int_0^t \xi(s)\eta(s)ds} \right\rangle = \exp\left\{ \frac{1}{2}\int_0^t dt_1 \int_0^t dt_2\, \delta(t_1-t_2)\xi(t_1)\xi(t_2) \right\} \end{cases}$$

確率分布関数 $\rho(x,t)$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(v\rho) = 0$$ 確率保存

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left\{ \left(\alpha(x,t) + \beta(x,t)\eta(t) \right) \rho(x,t) \right\} = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} = \mathbb{L}(t)\rho}$$ stochastic Liouville eq.

$$\mathbb{L}(t) = \mathbb{L}_0(t) + \mathbb{L}_1(t)$$

$$\equiv \underbrace{-\frac{\partial}{\partial x}\alpha(x,t)}_{\text{決定論}} \underbrace{- \frac{\partial}{\partial x}\beta(x,t)\eta(t)}_{\text{確率論}}$$

形式解

$$\rho(x,t) \equiv T e^{\int_0^t \mathbb{L}(t_1)dt_1} \rho(x,0)$$

$$= \exp_+\left\{ \int_0^t (\mathbb{L}_0(t_1) + \mathbb{L}_1(t_1)) \right\} \rho(x,0)$$

where we set $\mathbb{V}_0(t) = \exp_+ \int_0^t \mathbb{L}_0(t_1)dt_1$ とおく

$$= \mathbb{V}_0(t) \exp_+\left\{ \underbrace{\int_0^t \mathbb{V}_0^{-1}(t_1)\mathbb{L}_1(t_1)\mathbb{V}_0(t_1)dt_1}_{\mathbb{L}_1\text{の相互作用表示}} \right\} \rho(x,0)$$

確率分布

$$p(x,t) = \langle \rho(x,t) \rangle_\eta$$

$$= \mathbb{V}_0(t) \left\langle \exp_+ \left\{ \int_0^t \mathbb{V}_0^{-1}(t_1)\, \mathbb{L}_1(t_1)\, \mathbb{V}_0(t_1)\, dt_1 \right\rangle p(x,0)$$

$$\left\langle \exp_+ \left\{ -\int_0^t \underbrace{\mathbb{V}_0^{-1}(t_1) \frac{\partial}{\partial x} \beta(x,t_1)\, \mathbb{V}_0(t_1)}_{Q(t_1)} \quad \eta(t_1)\, dt_1 \right\rangle$$

Gaussianの性質である事に注意。
(演算子)

$$= \exp_+ \left\{ \int_0^t dt_1 \int_0^{t_1} dt_2\, \gamma(t_1-t_2)\, Q(t_1)\, Q(t_2) \right\}$$

$\gamma(t_1-t_2) = 2\varepsilon\, \delta(t_1-t_2)$ の場合.

$$= \exp_+ \left\{ \varepsilon \int_0^t Q^2(t_1)\, dt_1 \right\}$$

$$\therefore\ p(x,t) = \mathbb{V}_0(t)\, \exp_+ \left\{ \varepsilon \int_0^t Q^2(t_1)\, dt_1 \right\} p(x,0)$$

tで微分して.

$$\frac{\partial p(x,t)}{\partial t} = \overbrace{\mathbb{V}_0'(t)}^{\mathbb{L}_0(t)\mathbb{V}_0(t)} \exp_+ \left\{ \varepsilon \int \cdots\cdots \right\} p(x,0)$$

$$+\ \mathbb{V}_0(t)\, \varepsilon\, Q^2(t_1)\, \exp_+ \{ \varepsilon \cdots\cdots \} p(x,0)$$

$$= \left\{ \mathbb{L}_0(t) + \underbrace{\varepsilon\, \mathbb{V}_0(t) \left(\mathbb{V}_0^{-1}(t) \frac{\partial}{\partial x} \beta\, \mathbb{V}_0(t) \right)^2 \mathbb{V}_0^{-1}(t)}_{\varepsilon \frac{\partial}{\partial x}\beta \frac{\partial}{\partial x}\beta} \right\} p(x,t)$$

$$\boxed{\frac{\partial p(x,t)}{\partial t} = \left\{ -\frac{\partial}{\partial x} \alpha(x,t) + \varepsilon \frac{\partial}{\partial x} \beta(x,t) \frac{\partial}{\partial x} \beta(x,t) \right\} p(x,t)}$$

$$\frac{\partial f(x,t)}{\partial t} = \Omega(x,t) f(x,t)$$

$$f(x, t+\Delta t) = f(x,t) + \int_t^{t+\Delta t} dt_1\, \Omega(x,t_1) f(x,t_1)$$

$$= \left(1 + \int_t^{t+\Delta t} dt_1\, \Omega(x,t_1) + \int_t^{t+\Delta t} dt_1 \int_t^{t_1} dt_2\, \Omega(x,t_1)\Omega(x,t_2) + \cdots \right) f$$

$\xrightarrow{\text{平均}}$

(i) $\displaystyle\int_t^{t+\Delta t} dt_1 \langle \Omega(x,t_1) \rangle$

(ii) $\displaystyle\int_t^{t+\Delta t} dt_1 \int_t^{t_1} dt_2 \langle \Omega(x,t_1)\Omega(x,t_2) \rangle$ -

$\boxed{\Delta t\text{の1次までをしらべる}}$

Langevin $\dot{u} = \frac{1}{m}R(t)$ ×

① $\Omega = -\frac{\partial}{\partial x} U(t)$ なら

(i) $\Rightarrow 0$

(ii) $\displaystyle = \int_t^{t+\Delta t} dt_1 \int_t^{t_1} dt_2 \underbrace{\langle U(t_1)U(t_2) \rangle}_{2D\delta(t_1-t_2)} \frac{\partial^2}{\partial x^2}$

$= \Delta t\, D \frac{\partial^2}{\partial x^2}$

$U(t)$ が Gaussian Process
$\Rightarrow$ Diffusion eq.

(iii) Ω の odd $\to 0$.

(iv) Ω の even $(2n) \to (\Delta t)^n$

② $\Omega = \frac{\partial}{\partial u}\left(\gamma U - \frac{R}{m}\right)$ $\left\{\begin{array}{l} \text{つまり}\quad \dot{x} = U \\ \dot{U} = -\gamma U + \frac{1}{m}R(t) \end{array}\right.$ Langevin

(i) $\Delta t \frac{\partial}{\partial u} \gamma U$

(ii) $\displaystyle \frac{1}{m^2} \int_t^{t+\Delta t} dt_1 \int_t^{t_1} dt_2 \underbrace{\langle R(t_1) R(t_2) \rangle}_{2m\gamma kT\delta(t_1-t_2)} + O(\Delta t^2)$ ← むしする

$= \Delta t \frac{\gamma kT}{m} \frac{\partial^2}{\partial u^2}$

$$\frac{\partial P(U,t)}{\partial t} = \frac{\partial}{\partial U}\left(\gamma U + \frac{\gamma kT}{m}\frac{\partial}{\partial U}\right) P(U,t)$$

Fokker-Planck eq.

ここで $\gamma \to 0$ としても Diffusion にはいかない.

$$\boxed{\begin{array}{l}\text{広義の Riccati eq.}\\[4pt] \dfrac{dy}{dx} + p(x)y + Q(x)y^2 = R(x)\end{array}}$$

・$Q(x)y(x) = \dfrac{\frac{dU}{dx}}{U(x)}$ とおいて.

$$\to\ y = \frac{U'}{QU}, \quad y' = \frac{1}{Q^2U^2}\{U''UQ - UU'Q' - U'^2Q\}$$

を代入

$$\frac{1}{Q^2U^2}\{U''UQ - UU'Q' - U'^2Q\} + P\frac{U'}{QU} + \cancel{Q\frac{U'^2}{Q^2U^2}} = R$$

$$\frac{1}{QU}\left\{U'' - \frac{Q'}{Q}U' + PU'\right\} = R$$

$$\frac{1}{QU}\left\{U'' + \left(P - \frac{Q'}{Q}\right)U'\right\} = R.$$

$$U'' + \left(P - \frac{Q'}{Q}\right)U' = RQU$$

$$\boxed{\begin{array}{c}U'' + \left(P - \dfrac{Q'}{Q}\right)U' - RQU = 0\\[4pt] \text{U に関する 2 階の同次線型 eq.}\end{array}}$$

今 $R=1$, $P = \dfrac{Q'}{Q}$ とおると

$$y' + \frac{Q'}{Q}y - Qy^2 = 1 \iff U'' + QU = 0$$

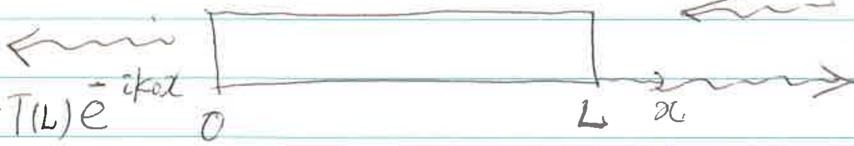
すると $\psi'' + k^2\psi = 0 \iff y' + \dfrac{2k'}{k}y - k^2y^2 = 1$

$$ky' + 2k'y - k^3y^2 = k$$

Invariant embeding

$\psi(x) = e^{-ik_0(x-L)} + R(L)\, e^{ik_0(x-L)}$ この region

$\psi(x) = T(L)\, e^{-ik_0 x}$ この region



$$\frac{d^2\psi}{dx^2} + K^2(x)\psi = 0 \qquad \begin{cases} K^2(x) \equiv k^2(x) = \frac{2m}{\hbar^2}[E - V(x)], & 0 \le x \le L \\ K^2(x) \equiv k_0^2 = \frac{2m}{\hbar^2}E, & x<0, \text{ or } x>L \end{cases}$$

$$\overbrace{|R(L)|^2}^{r} + |T(L)|^2 = 1,$$

$m = \hbar = 1$ とする

境界値問題 ⇒ 初期値問題

$$\boxed{ik_0 \frac{dR(L)}{dL} = V(L)(1+R^2(L)) - 2[k_0^2 - V(L)]R(L)}$$

Original 問題の3の導出

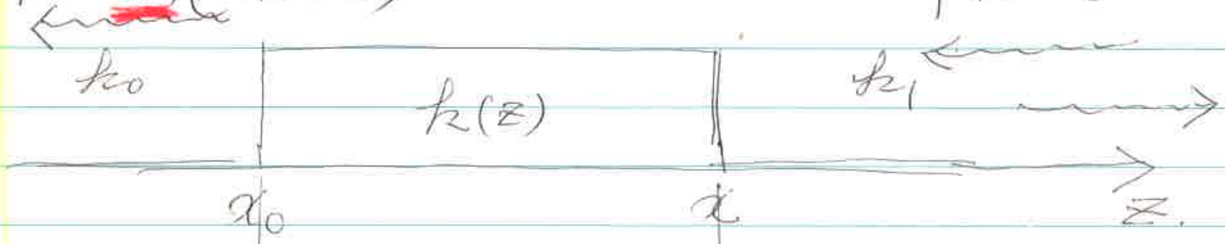
G.G. Stokes → Chandrasekhar ('60) ⇒ Bellman + Wing ('76)

元の問題

$$\frac{d^2\psi(z)}{dz^2} + K^2(z)\psi(z) = 0$$

$\psi(z) = \tau(x)\, e^{-ik_0(z-x_0)}$

$\psi(z) = e^{-ik_1(z-x)} + \rho(x)\, e^{ik_1(z-x)}$



$k_1 = k_0$ としてよい.

$$\begin{cases} u(z) \equiv \frac{1}{2ik_0}(\psi'(z) + ik_0\psi(z)) \\ v(z) \equiv \frac{-1}{2ik_0}(\psi'(z) - ik_0\psi(z)) \end{cases}$$

$v(z)$

$u(z) = e^{ik_0 z}$.

$$u'(z) = \left[\frac{1}{2ik_0^2}\right]\left\{-k_0\left(k^2(z) + k_0^2\right)u(z) - k_0\left(k^2(z) - k_0^2\right)v(z)\right\}$$

$$-v'(z) = \left[\frac{1}{2ik_0^2}\right]\left\{-k_0^2\left(k^2(z) - k_0^2\right)u(z) - k_0\left(k^2(z) + k_0^2\right)v(z)\right\}$$

境界条件

$$\begin{cases} \psi(x_0) = \tau(x), & \psi'(x_0) = -ik_0\tau(x) \\ \psi(x) = 1 + \rho(x), & \psi'(x) = -ik_0\left(1 - \rho(x)\right) \end{cases}$$

$$\Rightarrow \quad u(x_0) = 0, \quad v(x) = 1.$$

初期値問題

Eq. Riccati
二次の非線形

$$\begin{cases} \dfrac{d\rho}{dx} = \dfrac{-1}{2ik_0}\left\{\left[k^2(x) - k_0^2\right] + 2\left[k^2(x) + k_0^2\right]\rho(x) + \left[k^2(x) - k_0^2\right]\rho^2(x)\right\} \\ \rho(x_0) = 0. \end{cases}$$

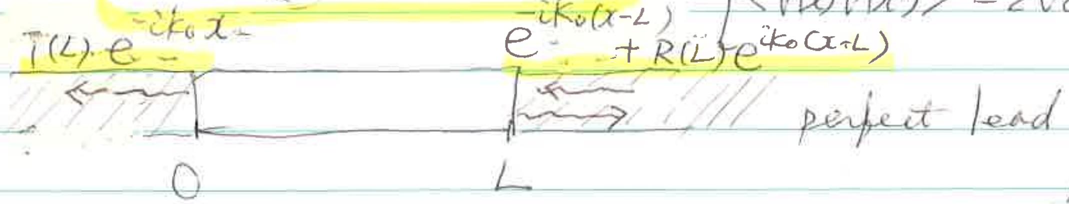
$$\begin{cases} \dfrac{d\tau}{dx} = \dfrac{-1}{2ik_0}\left\{\left[k(x) + k_0^2\right] + \left[k^2(x) - k_0^2\right]\rho(x)\right\}\tau(x) \\ \tau(x_0) = 1 \end{cases}$$

Randomness の有無に関らずつかえる!!

$$\frac{d\rho}{dr} = \frac{2r(1-r^2) - r^2(-2r)}{(1-r^2)^2} = \frac{2r}{(1-r^2)^2} = 2(1+\rho)^2\sqrt{\frac{\rho}{\rho+1}}$$

$$\boxed{H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)}$$

$$\begin{cases} \langle V(x)\rangle = 0 \\ \langle V(x)V(x')\rangle = 2V_0^2\,\delta(x-x') \end{cases}$$



$0 < x < L$

$$\frac{d^2\psi}{dx^2} + k^2(x)\psi = 0 \qquad \begin{cases} k^2(x) = k^2(x) = \frac{2m}{\hbar^2}(E - V(x)) \\ k^2(x) = k_0^2 = \frac{2m}{\hbar^2}E & x<0,\ x>L \end{cases}$$

境界値問題 ⟶ 初期値問題 (invariant imbedding)

2階線形 ⟺ Riccati equation (1階常微分 eq.) でも non-linear
線形微分eq.

常微分eq.より

$$\boxed{ik_0\frac{dR}{dL} = V(L)(1+R^2) - 2(k_0^2 - V(L))R} \quad — *$$

(complex)

$R = re^{i\theta}$ として non-linear coupled stochastic differential eqs.

$$\rho = \frac{r^2}{1-r^2} = \ldots$$

(real)
$$\begin{cases} \dfrac{d\rho}{dL} = -\dfrac{2V(L)}{k_0}\sin\theta\sqrt{\rho(\rho+1)} \\ \dfrac{d\theta}{dL} = 2k_0 - \dfrac{V(L)}{k_0}\left(2 + \cos\theta\,\dfrac{2\rho+1}{\sqrt{\rho(\rho+1)}}\right) \end{cases}$$

$(1+\rho)r^2 = \rho$

$r^2 = \dfrac{\rho}{1+\rho}$

$1 - r^2 = \dfrac{1}{1+\rho}$

stochastic Liouville eq.　分布 $Q(\rho, \theta, L)$

$$\frac{\partial Q}{\partial L} + (\nabla\cdot v)Q = 0 \qquad P(\rho,\theta,L) = \langle Q(\rho,\theta,L)\rangle_v$$

$$\frac{\partial P}{\partial L} = -\frac{\partial}{\partial\theta}\left\langle Q\left[2k_0 - \frac{V(L)}{k_0}\left(2 + \cos\theta\,\frac{(2\rho+1)}{\sqrt{\rho(\rho+1)}}\right)\right]\right\rangle_v$$

$$+ \frac{\partial}{\partial\rho}\left\langle Q\,\frac{2V}{k_0}\sin\theta\sqrt{\rho(\rho+1)}\right\rangle_v.$$

$L \gg \xi \qquad P(\rho,\theta,L) = W(\rho,L)\,\underbrace{P_1^s(\theta)}_{\text{stationary distribution}}$

◎本当かな.

(i) $P_1^s(\theta) = \dfrac{1}{2\pi}$ (random phase app.)

$\longrightarrow \quad \dfrac{\partial W(\rho,l)}{\partial l} = \dfrac{\partial}{\partial \rho}\left\{(\rho^2+\rho)\dfrac{\partial W}{\partial \rho}\right\}, \qquad \underline{l = L/\xi}$

$u = \ln(1+\rho) \qquad\qquad \xi = \dfrac{k_0^2}{2V_0^2}$

$\longrightarrow$

$\dfrac{\partial W_1(u)}{\partial l} = -\dfrac{\partial W_1}{\partial u} + \dfrac{\partial^2 W_1}{\partial u^2}$

$\rho \ll 1$のとき

$\dfrac{\partial W(\rho)}{\partial l} = \dfrac{\partial}{\partial \rho}\rho\dfrac{\partial W}{\partial \rho}$

$W(\rho) = \dfrac{e^{-\rho/l}}{l}$

$\longrightarrow \quad \boxed{W_1(u,l) = \dfrac{1}{\sqrt{2\pi\Delta \overline{U^2(l)}}}\exp\left\{-\dfrac{(u-\overline{u})^2}{2\Delta \overline{U^2(l)}}\right\}}$

$\overline{u(l)} = l, \quad \Delta\overline{U^2(l)} = l \quad \Leftrightarrow$ one parameter scaling.

(ii) ◎をつかい stationary distribution を出す

$-2k_0 P_1 + \dfrac{4V_0^2}{k_0^2}(1+\cos\theta)\dfrac{\partial}{\partial\theta}(1+\cos\theta)P_1 = \text{const}$

$P_1^s(\theta) = \lim_{\varepsilon\to 0} \underbrace{N}_{\text{normalization}} \underline{P(\theta,\varepsilon)}$

$\qquad\qquad \| $

$e^{-\phi(\theta)}\displaystyle\int_\theta^{\theta+2\pi} d\theta' \frac{e^{\phi(\theta')}}{\left[\varepsilon + \left(\frac{4V_0^2}{k_0^2}\right)(1+\cos\theta')^2\right]}$

$\phi(\theta) = \displaystyle\int_\pi^\theta d\theta' \frac{\left[2k_0 + \left(\frac{4V_0^2}{k_0^2}\right)\sin\theta'(1+\cos\theta')\right]}{\left[\varepsilon + \quad '' \quad\right]}$

一般的でない

仙音光

$\rho << 1$ つまり $\ell >> \xi$ のとき

$\frac{\partial W(\rho)}{\partial \ell} = \frac{\partial}{\partial \rho}\rho\frac{\partial W(\rho)}{\partial \rho}$ の解として $W(\rho) = \frac{e^{-\rho/\ell}}{\ell}$ をもつ

<check>

左辺 $\frac{1}{\ell}e^{-\rho/\ell}(-\rho)(-\frac{1}{\ell^2}) + (-\frac{1}{\ell^2})e^{-\rho/\ell}$

$= \frac{1}{\ell^2}e^{-\rho/\ell}\{\frac{\rho}{\ell} - 1\}$

右辺 $\frac{\partial}{\partial \rho}\{\rho\frac{e^{-\rho/\ell}}{\ell}(-\frac{1}{\ell})\} = -\frac{1}{\ell^2}\{e^{-\rho/\ell} + \rho e^{-\rho/\ell}(-\frac{1}{\ell})\}$

$= \frac{e^{-\rho/\ell}}{\ell^2}\{\frac{\rho}{\ell} - 1\}$ // OK

これは

$S[W(\rho)] = -\int W(\rho)\ln W(\rho)\,d\rho$ を

$\langle \rho \rangle_\ell = \ell$ (Ohm's law) で maximize する

当然 $\langle \rho \rangle_\ell = \int_0^\infty W(\rho)\rho\,d\rho = \int_0^\infty \frac{e^{-\rho/\ell}}{\ell}\rho\,d\rho = \ell$ OK

$L^{d-2} \to L^{-1}$

$g \sim L^{d-2}$ $\qquad$ $g \propto L^{-1}$

$\rho \propto L$

乱流モデル計算で導入.

Novikov の定理 ('65)

$$\langle f_i(x) f(x') \rangle$$

$$\langle f_i(x, t+\tau) f_k(x', t) \rangle = F_{ik}(x, x') \delta(\tau)$$

$$F_{ik}(x-x')$$

Gaussian Random function

$$\boxed{\langle f_i(x,t) R[f] \rangle = \int F_{ik}(x-x') \left\langle \frac{\delta R[f]}{\delta f_k(x';t) dx' dt} \right\rangle dx'}$$

(証)

$$R_{i_1 \cdots i_n}^{(n)}(s_1 \cdots s_n)$$

$$R[f] = R[0] + \sum_{n=1}^{\infty} \frac{1}{n!} \int \cdots \int \frac{\delta^n R[f]}{\delta f_{i_1}(s_1) ds_1 \, \delta f_{i_2}(s_2) ds_2 \cdots \delta f_{i_n}(s_n) ds_n} \Bigg|_{f=0}$$

$$\times f_{i_1}(s_1) \cdots f_{i_n}(s_n) ds_1 ds_2 \cdots ds_n \quad — (*)$$

まず

$$\langle f_i(s) R[f] \rangle = \sum_{n=1}^{\infty} \frac{1}{n!} \int \cdots \int R_{i_1 \cdots i_n}^{(n)}(s_1 \cdots s_n) \langle f_i(s) f_{i_1}(s_1) \cdots f_{i_n}(s_n) \rangle ds_1 \cdots ds_n$$

Gaussian の条件

$$\sum_{\alpha=1}^{n} \langle f_i(s) f_{i_\alpha}(s_\alpha) \rangle \langle f_{i_1}(s_1) \cdots f_{i_{\alpha-1}}(s_{\alpha-1}) f_{i_{\alpha+1}}(s_{\alpha+1}) \cdots \rangle$$

$$= \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \int F_{i i_1}(s, s_1) \left[\int \cdots \int R_{i_1 i_2 \cdots i_n}^{(n)}(s_1, s_2, \ldots, s_n) \langle f_{i_2}(s_2) \cdots f_{i_n}(s_n) \rangle ds_2 \cdots \right] ds_1$$

// 等しい

一方 (*) から direct に

$$\frac{\delta R}{\delta f_k(s') ds'} = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \int \cdots \int R_{k i_2 \cdots i_n}^{(n)}(s', s_2, \ldots, s_n) f_{i_2}(s_2) \cdots f_{i_n}(s_n) ds_2 \cdots ds_n$$

故に

$$= \int F_{i i_1}(s, s_1) \left\langle \frac{\delta R}{\delta f(s_1) ds_1} \right\rangle ds_1$$

§ 2 Novikov identity — ＊.

$$\langle V(L')\, Q(\rho, \theta, L) \rangle_V$$

$$= \int_0^L \underbrace{\langle V(L')\, V(L'') \rangle}_{= 2V_0^2\,\delta(L'-L'')} \left\langle \frac{\delta Q}{\delta V(L'')} \right\rangle dL'$$

$$= \not{2} V_0^2 \, \frac{1}{\not{2}} \left\langle \frac{\delta Q}{\delta V(L)} \right\rangle$$

$\frac{\delta}{\delta V}\left(\frac{\partial P}{\partial L}\right)$? — ＊

$$= \frac{\partial}{\partial \theta}\left\{ \frac{1}{k_0}\left(2 + \cos\theta \frac{2\rho+1}{\sqrt{\rho(\rho+1)}} \right) P \right\} + \frac{\partial}{\partial \rho}\left\{ \frac{2}{k_0} \sin\theta \sqrt{\rho(\rho+1)}\, P \right\}$$

$$\Rightarrow \frac{\partial P}{\partial L} = -2k_0 \frac{\partial P}{\partial \theta} + \frac{V_0^2}{k_0^2}\frac{\partial}{\partial \theta}\left\{ A(\rho,\theta)\left[\frac{\partial}{\partial \theta}\left(A(\rho,\theta)P\right) + \frac{\partial}{\partial \rho}\left(B(\rho,\theta)P\right) \right] \right\}$$

$$+ \frac{V_0^2}{k_0^2}\frac{\partial}{\partial \rho}\left\{ B(\rho,\theta)\left[\frac{\partial}{\partial \theta}\left(A(\rho,\theta)P\right) + \frac{\partial}{\partial \rho}\left(B(\rho,\theta)P\right) \right] \right\} \text{ — ④＊}$$

$$A(\rho, \theta) = 2 + \cos\theta \frac{2\rho+1}{\sqrt{\rho(\rho+1)}}$$

$$B(\rho, \theta) = 2\sin\theta\sqrt{\rho(\rho+1)}$$

dimensionless parameter

$k_0 L$, $\frac{V_0^2}{k_0^3} \equiv \Delta$.

Δ 小, weak disorder

Δ 大 strong disorder

marginal distribution

$$W(\rho) = \int P(\rho, \theta, L)\, d\theta$$

thermodynamic limit $L \to \infty$. $\Rightarrow$ $\rho \gg 1$, $r^2 \simeq 1$.

$$\frac{d\theta}{dL} = 2k_0 - \frac{2V}{k_0}(1+\cos\theta)$$

Langevin → FP の関係をつかう (5)型とした.

$$\Rightarrow ⑨\ \frac{\partial P_1(\theta, L)}{\partial L} = -2k_0 \frac{\partial P_1}{\partial \theta} + \left(\frac{4V_0^2}{k_0^2}\right)\frac{\partial}{\partial \theta}\left[(1+\cos\theta)\frac{\partial}{\partial \theta}(1+\cos\theta) P_1 \right]$$