Delocalization and Classicalization of Quantum Wavepacket in Kicked Anderson Model

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We consider quantum diffusion of the initially localized wavepacket in one-dimensional kicked disordered system with classical coherent perturbation. The wavepacket localizes in the unperturbed kicked Anderson model. However, the wavepacket gets delocalized even by coupling with monochromatic perturbation. We call the state "dynamically delocalized state." It is numerically shown that the delocalized wavepacket spreads obeying diffusion law, and the perturbation strength dependence of the diffusion rate is given. The sensitivity of the delocalized state is also shown by the time-reversal experiment after random change in phase of the wavepacket.

KEYWORDS: localization, delocalization, kick, disordered, classicalization, diffusion

1. Introduction

Recently, we have found an interesting property in Anderson model with coherently time-dependent perturbation.[1–7] In one-dimensional disordered system (1DDS), it is well known that almost all eigenstates are localized. The quantum diffusion of initially localized wavepacket in the 1DDS is suppressed at the localization length, as a result of the interference between scattering waves.[8] When classical coherent perturbation consisting of some frequency components makes the localized state *delocalized*, we called the state *dynamically delocalized state*.[3,4] Moreover, when we couple a harmonic oscillator with the delocalized state in the perturbed Anderson model, the energy, prepared in an excited state, has been irreversibly transferred to the oscillator.[2–4,7] The similar localization and delocalization phenomena can be observed in kicked Anderson model with coherently time-dependent perturbation.[9,10] Accordingly, instead of Anderson model we can use the kicked Anderson model to investigate the localization and the delocalization phenomena. Generally the kicked system is convenient to save CPU time for long-time numerical simulation. In the present paper, we investigate the delocalization and the sensitivity to the change of the phase as perturbations from other degrees of freedom (DOF) is introduced in the kicked 1DDS.

The outline of the present paper is as follows. In Sec. 2 model system investigated is introduced. In Sec. 3, we investigate the wavepacket dynamics in coherently perturbed kicked Anderson model. In Sec. 4, we give numerical result for time-reversal experiments after random change in phase of the wavepacket in the dynamics. The last section is devoted to summary and discussion.

2. Model

We consider a parametrically perturbed kicked Anderson model to investigate delocalization phenomena in the 1DDS, which Hamiltonian is given as follows:

$$H^{\rm osc}(t) = T(\hat{p}) + \sum_n |n\rangle V(n,t)\langle n| \sum_m \delta(t-m),$$

$$V(n,t) = V(n)\left(1 + \frac{\epsilon}{\sqrt{L}}\sum_j^L \cos\omega_j t\right),$$

where $\hat{p}\,(=-i\partial/\partial x)$ is a shift operator and $V(n)$ is uniformly distributed on-site energy in the range $[-W, W]$. $T(\hat{p}) = 2(\cos\hat{p} - 1)$ is hopping terms between nearest neighbor sites. The frequency components of the classical coherent perturbation $\{\omega_j\}$ are taken to be mutually incommensurate numbers and the order is $O(1)$. The time evolution operator for s steps is given by,

$$U(s) = \prod_{k=1}^{s} e^{-iT(\hat{p})/2\hbar} e^{-iV(n,k)/\hbar} e^{-iT(\hat{p})/2\hbar}.$$

The value of the wave function is determined in the middle of the two successive kicks, and the periodic boundary conditions are assumed.

3. Dynamical Delocalization

We show time dependence of the mean square displacement (MSD), $m_2(t) = \langle\Psi(t)|(n-n_0)^2|\Psi(t)\rangle$, of initially localized wavepacket $\Psi(n, t = 0) = \delta_{n,n_0}$ in the parametrically perturbed kicked Anderson model. It has been numerically shown that in the unperturbed case ($L = 0$), diffusion of the wavepacket is suppressed and localized around certain localization length.[9,10] As seen in Fig. 1 the wavepacket is delocalized even by the monochromatic perturbation ($L = 1$), and we can call the state *dynamically delocalized state* in a sense that any stochastic perturbation is not imposed on the system.[3,4] Note that the axes of Fig. 1(b) are in log-scale. Accordingly when the perturbation strength is small, it seems that the diffusion is not a normal diffusion: subdiffusions are well observed. It seems that the subdiffusion gradually turns to normal diffusion as the perturbation strength increases to 0.2 from 0.1. It is, however, difficult to judge whether the critical value ϵ_c dividing the subd-

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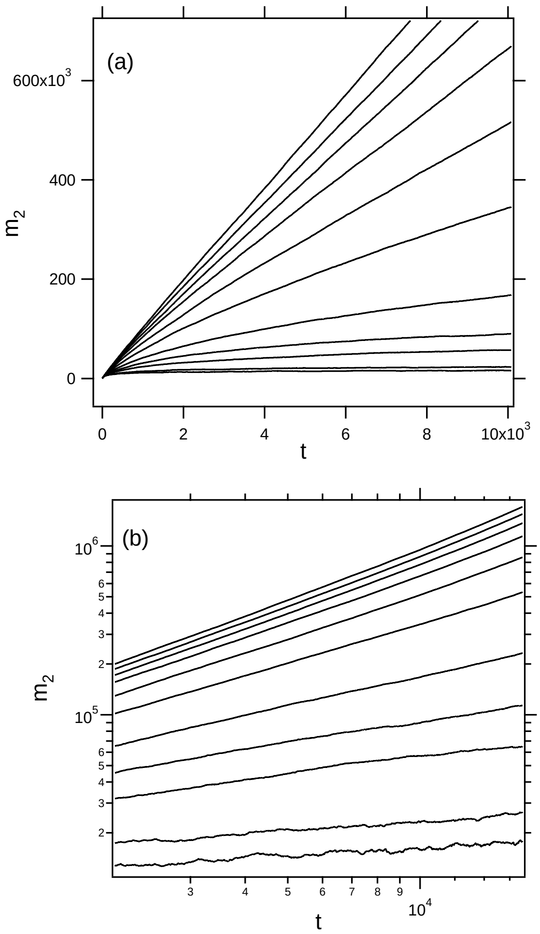

Fig. 1. (a) Time dependence of MSD in kicked Anderson model with monochromatic perturbation ($L = 1$) at some perturbation strength ϵ, and (b) the log–log plots. The system and ensemble sizes are 2^{14} and 30 respectively. ($W = 0.5$, $\hbar = 0.125$.)

iffusion and normal diffusion exists or not by the numerical data. We can characterize the subdiffusion by the index α in a form, $m_2 \sim t^{\alpha}$. Figure 2(b) shows the index α as a function of the perturbation strength ϵ. The exponent α approaches unity from below with increasing ϵ. The diffusion rate D estimated by $m_2 \simeq Dt$ in Fig. 1(a) is given in Fig. 2(a). Figure 3 shows time-dependence of the ensemble-averaged probabilistic function $P(n,t) = \langle|\Psi(n,t)|^2\rangle$. The functional form of the distribution function approaches Gaussian function as increase of the strength ϵ, which corresponds to a solution in diffusion equation in the stochastic process. We can regard the delocalization as a kind of "classicalization" of the quantum wavepacket, which is caused by the coupling with the other DOF. It is found that the number of DOF for the "classicalization" is three based on the autonomous transformation of the time-dependent Hamiltonian.

When the number of the frequency components of the perturbation increases ($L = 2, 3$), normal diffusion brings about even in the much smaller perturbation strength than monochromatically perturbed case.[10] The details will be reported elsewhere.[10] In the next section we focus on only the monochromatically perturbed cases.

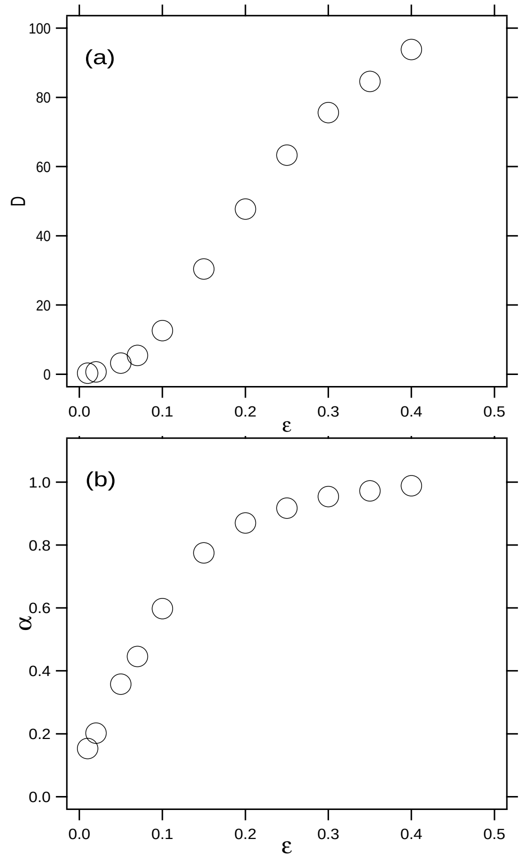

Fig. 2. Perturbation strength dependence of (a) diffusion rate D and (b) power index α estimated by data in Fig. 1.

It is worth noting that the delocalization can be brought about in a quantum system composed of three DOF when we map the time-dependent Hamiltonian on autonomous Hamiltonian.[11] We have already reported that in the perturbed Anderson model, not kicked Anderson model, more than two frequency components are necessary to observe the delocalized phenomena.[3,4] Then total number of DOF was three. It correspond to monochromatically perturbed kicked Anderson model concerning the number of DOF.

4. Time Irreversibility

In this section, we show the result for time-reversal experiments after random change in phase of the wavepacket in the dynamics of the unperturbed and the monochromatically perturbed kicked Anderson model, comparing with results in periodic system.

First the system evolves by the time-evolution operator U, until $t = T$. At $t = T$ a perturbation $\hat{P}$ is applied for the wavepacket. We used the random change in phase of the wavepacket $\Psi(n,T)$ at each site n. Accordingly the perturbation does not change the probability amplitude $|\Psi(n,T)|^2$. After applying the perturbation $\hat{P}$, we evolve the state by the unitary operator $U(T-t')$ until $t' = T$ as follows

$$\Psi(t'+T) = U(T-t')\hat{P}U(T)\Psi(0).$$

In the concrete, we change the state of the packet at

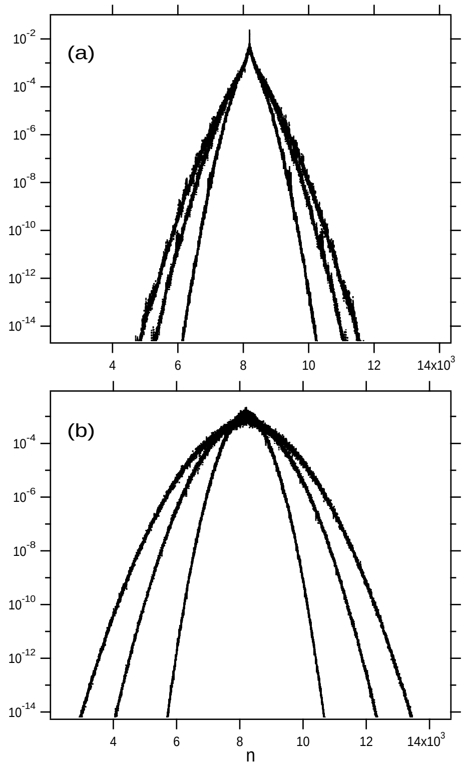

Fig. 3. Some snapshots of the ensemble averaged probability distribution function of the monochromatically perturbed kicked Anderson model ($L = 1$) with (a) $\epsilon = 0.1$, (b) $\epsilon = 0.4$. ($t = 1000$, 3000, 5000.)

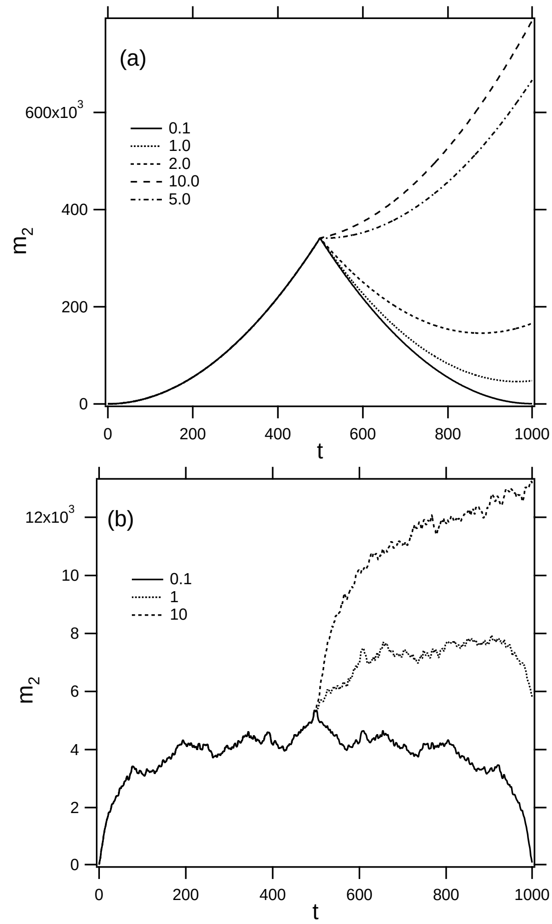

Fig. 4. Time reversal experiments for various phase-change strength a in (a) binary periodic and (b) disordered systems without perturbation($L = 0$). The system size is 2^{14} and we used a sample. ($W = 0.5$, $\hbar = 0.125$, $T = 500$.)

$t = T$ by an operation $\hat{P}$,

$$\Psi(n, T) \to e^{ia\xi_n}\Psi(n, T),$$

where ξ_n ($\in [-1, 1]$) is a random change in phase at each site n and a is the change-strength. We monitor the time dependence of the MSD for the total process.

Figure 4 shows the results for various phase-change strength a in binary periodic case and disordered case without perturbation ($L = 0$). In both cases as increase of the strength a the state does not return to the initial state and begins to diffuse. If we continue the simulation beyond the time $t = 2T$, the wavepacket ballistically diffuses in the periodic system and is localized in the disordered system.

Figure 5 shows the results of the time reversal experiments for various random change in phase strength a in periodic system with monochromatic perturbation ($L = 1$, $\epsilon = 0.2$). In a case $a = 0.1$, the state almost returns to the initial state although the wavepacket spreads over the system. As increase of the value a, it becomes difficult to return to the initial state.

Figure 6 shows the results of the time reversal experiments at a fixed value $a = 0.1$ in disordered system with the monochromatic perturbation. It is found that different from the periodic system, even for small perturbation strength irreversibility is built in the system although the wavepacket does not spread over the system.

Totally we can say that the dynamically delocalized state in the disordered system with coherent perturbation has much potential for irreversibility, and has ability to be entangled with the other quantum state when compared to cases in periodic system. The same idea based on the time irreversibility has been used to investigate the "classicalization" for quantum chaos systems.[12)]

5. Summary and Discussion

We numerically investigated delocalization of initially localized quantum wavepacket in kicked Anderson model with coherent time-dependent perturbation. In the monochromatically perturbed cases, diffusive behavior maintains within the time scale accessible by the numerical simulation. The diffusion process is anomalous for small perturbation strength $\epsilon \ll 1$, in which the MSD increases as $m_2(t) \propto t^\alpha$ ($\alpha < 1$). As increase of the perturbation strength, the subdiffusion approach normal diffusion. We can regard the delocalization as a kind of "classicalization" by coupling with small number of DOF.

Next we performed time-reversal experiments for the dynamically delocalized states. In the dynamically delocalized states it is difficult to recover the initial memory by the time-reversal process after random change in phase of the wavepacket in a sense that amplitude of the wavepacket does not take the initial value. We can say that the dynamically delocalized states are sensitive

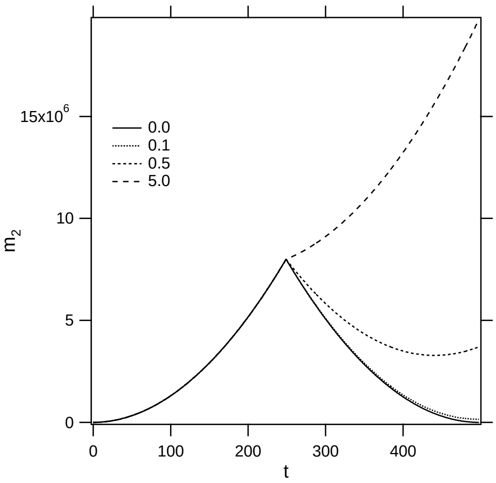

Fig. 5. Time reversal experiments for various phase-change strength a in a periodic system with the monochromatic perturbation ($L = 1$, $\epsilon = 0.2$, $T = 250$). The other parameters are the same as in Fig. 4.

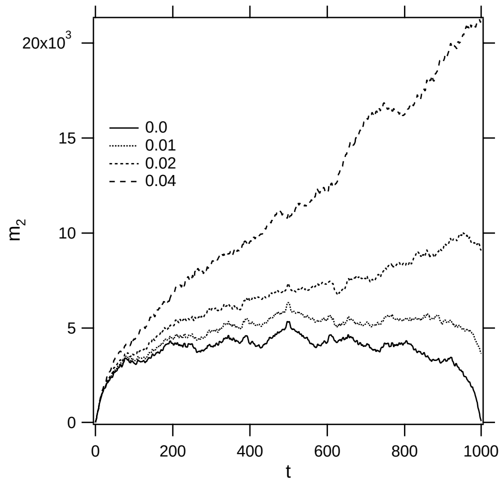

Fig. 6. Time reversal experiments for various perturbation strength ϵ in the disordered system with the monochromatic perturbation ($L = 1$, $a = 0.1$, $T = 500$). The other parameters are the same as in Fig. 4.

to the random change in phase of the wavepacket and this property is one of the remarkable properties different from quantum state in periodic systems. Moreover, the delocalized states become more sensitive to the random change in phase as increase of the strength ϵ. The more details will be given in our forthcoming paper.[10)]

Acknowledgments

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1) H. Yamada, K. Ikeda and M. Goda: Phys. Lett. A **182** (1993) 77.
2) H. Yamada and K. S. Ikeda: Phys. Lett. A **222** (1996) 76.
3) H. Yamada and K. S. Ikeda: Phys. Lett. A **248** (1998) 179.
4) H. Yamada and K. S. Ikeda: Phys. Rev. E **59** (1999) 5214.
5) H. Yamada and K. S. Ikeda: Phys. Rev. E **65** (2002) 046211.
6) H. Yamada: Physica B **316** (2002) 350.
7) H. Yamada: *Fundamental Aspects of Quantum Physics*, ed. L. Accardi and S. Tasaki (World Scientific, 2003) p. 80.
8) K. Ishii: Prog. Theor. Phys. Suppl. **53** (1973) 77.
9) H. Yamada: *Quantum Limits to the Second Law*, ed. D. P. Sheehan (AIP Conference Proceedings 643, 2002) p. 77.
10) H. Yamada and K. S. Ikeda: in preparation.
11) K. Ikeda: Ann. Phys. (N.Y.) **277** (1993) 1.
12) K. S. Ikeda: *Quantum Chaos Between Order and Disorder*, ed. G. Casati and B. Chirikov (Cambridge University Press, 1995) p. 147.