

## Localization and Delocalization in Kicked Anderson Model

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(Received August 17, 2002)

KEYWORDS: localization, delocalization, kick, disordered, diffusion

### 1. Introduction

Recently, we have found an interesting property in Anderson model with coherently time-dependent perturbation.<sup>1–4)</sup> In Anderson model, it is well-known that almost all eigenstates are localized and the quantum diffusion of initially localized wavepacket is suppressed at the localization length. When classical coherent perturbation make the localized state delocalized and we call the states "dynamically delocalized states". Moreover, when we couple a harmonic oscillator with the perturbed Anderson system in the delocalized state, the energy have been irreversibly transferred from the electron system to the oscillator in ground state.<sup>2,5–8)</sup>

In the present paper we investigate the localization phenomena in kicked Anderson model and delocalization caused by coherent external perturbation, comparing with the dynamical delocalization in Anderson model.<sup>2–4,7,9)</sup>

### 2. Models

The dynamics of kicked Anderson model is given by Floquet operator,

$$U = \exp\left(-\frac{i}{\hbar} \frac{T(\hat{p})}{2}\right) \exp\left(-\frac{i}{\hbar} V(x)\right) \exp\left(-\frac{i}{\hbar} \frac{T(\hat{p})}{2}\right),$$

where  $\hat{p} = -i\frac{\partial}{\partial x}$  and  $V(x)$  is uniformly distributed random on-site energy in the range  $[-W, W]$  and  $T(\hat{p}) = 2(\cos \hat{p} - 1)$  is hopping terms between nearest neighbor sites. The value of the wave function is determined in the middle of the between two successive kicks, and the periodic boundary condition is assumed. Note the form is equivalent to second order symplectic integration for time evolution of general quantum system. It is instructive to recall the relation between Harper model and kicked Harper model.<sup>10–12)</sup>

Furthermore, we consider parametrically perturbed kicked Anderson model to investigate delocalization phenomena, which Hamiltonian is given as follows:

$$H^{osc}(t) = T(\hat{p}) + V(x) \left\{ 1 + \frac{\epsilon}{\sqrt{L}} \sum_j^L \cos(\omega_j t + \phi_j) \right\} \\ \times \sum_m \delta(t - m),$$

where the frequency components of the classical coherent

perturbation  $\{\omega_j\}$  are taken to be mutually incommensurate numbers and the order is  $O(1)$ .

### 3. Numerical Results

As far as we know, the localization phenomena of the kicked Anderson model have not been investigated. In Fig.1 firstly we numerically show time dependence of the mean square displacement (MSD),  $m_2(t) = \xi^2(t) = \langle \Psi(t) | x^2 | \Psi(t) \rangle$ , of initially localized wave packet  $\Psi(x, t = 0) = \delta_{x, N/2}$  in the kicked Anderson model, where  $N$  is system size. It is very interesting whether a critical value  $W_c$  dividing exponentially localized states and delocalized states is present or not. More extensive numerical study is necessary to divide the conclusion.<sup>14)</sup> It seems that for small  $W$  the MSD grow up unlimitedly in this time scale. We, however, consider the strongly localized cases to investigate delocalization phenomena by coherent perturbation in this report. It found that in the localized regime diffusion of the packets are suppressed and exponentially localized. The localization length  $\xi$  is enhanced as the strength  $W$  of the fluctuation of the randomness, which is equivalent to strength of the kick, increases as  $\xi \propto W^{-2}$ .<sup>7)</sup> (See Fig.2.)

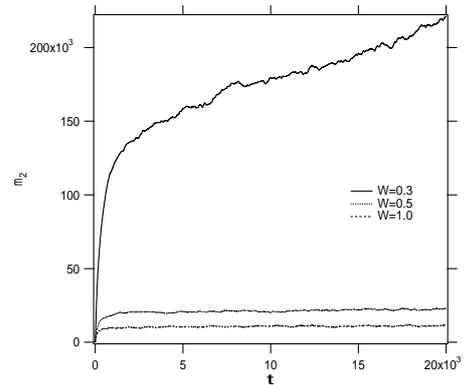


Fig. 1. Time dependence of MSD of kicked Anderson model with some potential strength  $W$ . The system and ensemble size are  $2^{14}$  and 30 respectively.  $\hbar = 0.125$ .

Next, we briefly summarize the delocalized phenomena in the kicked Anderson model with the classical coherent perturbation. As seen in Fig. 3 the wave packet is delocalized even in the monochromatically perturbed case ( $L = 1$ ), and we call the states "dynamically delocalized states" in a sense that any stochastic pertur-

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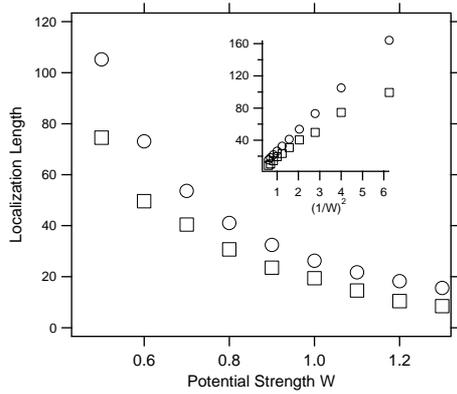


Fig. 2. Localization length as a function of potential strength  $W$  in the kicked Anderson model. As a reference localization length of band center of Anderson model are also shown by open squares.  $\hbar = 1.0$

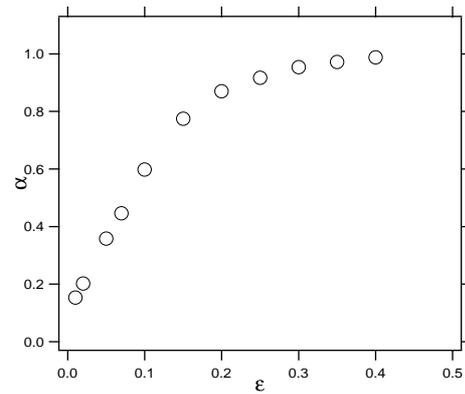


Fig. 4.  $\epsilon$ -dependence of the index  $\alpha$  estimated by data in Fig.3(a).

bation is not imposed on the system.<sup>4)</sup> The diffusion process is not general normal diffusion but anomalous diffusion characterized by index  $\alpha$  such as  $m_2(t) \propto t^\alpha$  for large-time scale. The  $\epsilon$ -dependence of the  $\alpha$  is shown in Fig.4. It is worth noting that the delocalization can be brought about in a quantum system composed of three degrees of freedom. The situation is same to a parametrically perturbed Anderson model.<sup>4)</sup> On the other hand, in "host-helper system" studied by Ikeda<sup>13)</sup> more than three frequency components are necessary to observe the delocalized phenomena.

#### 4. Summary

We investigated the delocalized phenomena in the kicked Anderson model with classical coherent perturbation. The delocalization could be observed even in monochromatically perturbed cases. We can regard the delocalization as "classicalization" of the wave packet by coupling with the other degrees of freedom.

#### Acknowledgements

This report is based on a collaboration with Professor K. S. Ikeda. The author would like to thank him for valuable suggestions and stimulating discussions about this study.

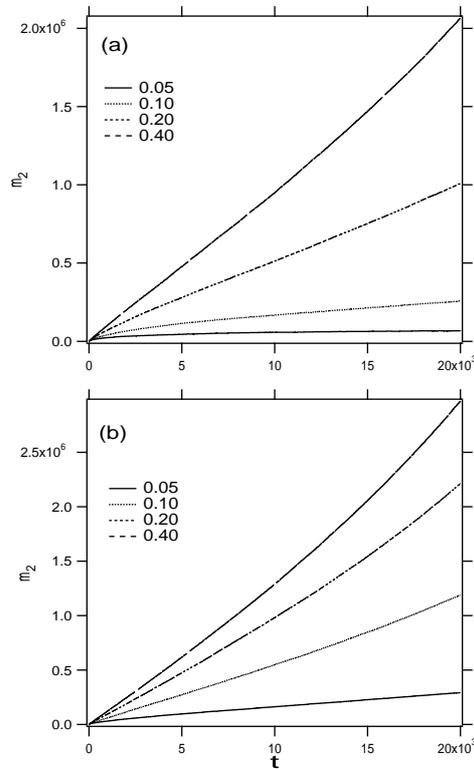


Fig. 3. Time dependence of MSD of kicked Anderson model with (a) monochromatic ( $L = 1$ ) and (b) dichromatic ( $L = 2$ ) perturbation. The other parameters are same to Fig.1.

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