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Localization in One-Dimensional Correlated Disordered System

Hiroaki YAMADA*

Department of Material Science and Technology, Faculty of Engineering,
Niigata University, Niigata 950-2181

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1. Introduction

The localization phenomena in one-dimensional (1D) disordered system have been extensively studied. It is well known that almost all the eigenstates are exponentially localized and the system has pure point energy spectrum under the presence of any disorder. Recently, the effect of the long range correlated disorder in the potential field on the localization has been reported by some groups.[1–5] Following tightly binding one-band Hamiltonian is used;

$$H = \sum_{n=1}^{N} |n > \epsilon_n < n| + \sum_{n}^{N} (|n >< n+1| + |n+1 >< n|),$$

where $\{\epsilon_n\}$ is the sequence of the onsite energy of electron with the correlated disorder.

Moura *et al.* showed that for long-range correlation with persistent increments, the power spectrum of the sequence is $S(f) \sim f^{-\alpha}$, Lyapunov exponent (inverse localization length) of the wave function vanishes within a finite range of energy values revealing the presence of an Anderson-like metal-insulator transition.[3] All one-electron states remain localized for $\alpha < 2$, but there is a finite range with extended eigenstates for $\alpha > 2$. On the other hand, Izrailev *et al.* showed mobility edge may exist in 1D potentials with correlated disorder in weak disorder limit even in $\alpha < 2$.[4] Furthermore, Yamada *et al.* discussed localization-delocalization transition of the electronic state of a special energy ($E = 0$) by means of modified Bernoulli model.[1,2,5] The statistical property of the sequence $\{\epsilon_n\}$ is generated by a modified Bernoulli map characterized by bifurcation parameter B. The correlation function $C(n) =< \epsilon_0 \epsilon_n >$ decays by inverse power-law depending on the value $B (\geq 3/2)$ as $C(n) \sim n^{-\frac{2-B}{B-1}}$ for large n. The power spectrum becomes $S(f) \sim f^{-\frac{2B-3}{B-1}}$ for small f. It has been reported that the stationary regime ($B < 2$) the system have pure point spectrum even in the correlated disorder cases.[1] The critical case $\alpha = 2$ in Brownian motion-like model by Moura *et al.* corresponds to a limit $B \to \infty$ in our model.

In this report we investigated Lyapunov exponent of the modified Bernoulli model in wide energy range for regime $3/2 < B < 3$ that corresponds to a regime, $0 < \alpha < 3/2$. We show numerically that a possibility of the presence of the mobility edge in non-stationary regime ($B \geq 2$) for weak disorder limit.[7]

2. Model

We briefly introduce a modified Bernoulli map and the statistical properties of the sequence. The map is

$$X_{i+1} = \begin{cases} X_i + 2^{B-1}(1-2b)X_i^B + b & (0 \leq X_i < 1/2) \\ X_i - 2^{B-1}(1-2b)(1-X_i)^B + b & (1/2 \leq X_i \leq 1), \end{cases}$$

where B is a bifurcation parameter which controls the correlation of the sequence. And the b stands for deterministic perturbation which is set $b = 10^{-12}$ only for $B \geq 2$ in this paper. For numerical simulation b is used in order to overcome difficulty that comes from non-stationary. Stationary is recovered by the perturbation though the essential property remains invariant for a long time $i < i_b$, where $i_b = (B-1)^{-1}(2b)^{(1-B)/B}$.[6] We use a sequence $\{\epsilon_n\}$ given by $\epsilon_n = (2X_n - 1)W$, where the W is strength of the fluctuation. Moreover, the sequence $\{X_i\}$ can be symbolized by the following rule:

$$\begin{cases} 0 \leq X_i < 1/2 \to \epsilon_i = -W \\ 1/2 \leq X_i < 1 \to \epsilon_i = W \end{cases}$$

It is analytically shown that the correlation function of the symbolized sequence decreases obeying power-law for large, $C(n) \sim n^{-\frac{2-B}{B-1}}$.[6] It is worth noting that in $B < 2$ the normalizable stationary distribution (invariant measure) exists, on the other hand in $B \geq 2$ the sequence becomes non-stationary and the normalizable measure does not exist when $b = 0$.[6] This property of the sequence strongly affects the convergence property of the Lyapunov exponents for the system.

3. Numerical Result

For large system size N, Lyapunov exponent is defined as $\gamma =< \frac{ln(|\phi_{N+1}|^2 + |\phi_N|^2)}{2N} >$, where ϕ_N is amplitude of the wave function at site N and $< ... >$ denotes ensemble average over different configuration. We restrict our numerical computation within $B \leq 3$, because it is difficult for large value of B to detect the convergent Lyapunov exponents.

* E-mail address: hyamada@cc.niigata-u.ac.jp

In Fig.1 we show plots of γ versus energy E for $B = 2.0$. We found that in weak disorder limit ($W << 1$) γ vanishes within a finite range of energy that reveal the presence of a phase of extended states near the center of the band ($E = 0$). We use a band center to investigate a delocalized state.

In Fig.2 we show γ versus B in the symbolic cases. Close to a bifurcation point ($B = 2.0$) γ approach zero for the all cases. Inset of the Fig.2 shows that for weak disorder limit $B-$dependence of γ rapidly change like a transition. Apparently we can say the slope of the decrease for $\gamma(B)$ as a function B change around $B = 2.0$. The more details of the numerical analysis will be given elsewhere.[7] The similar behavior is observed in cases of non-symbolic model. (See Fig.3.)

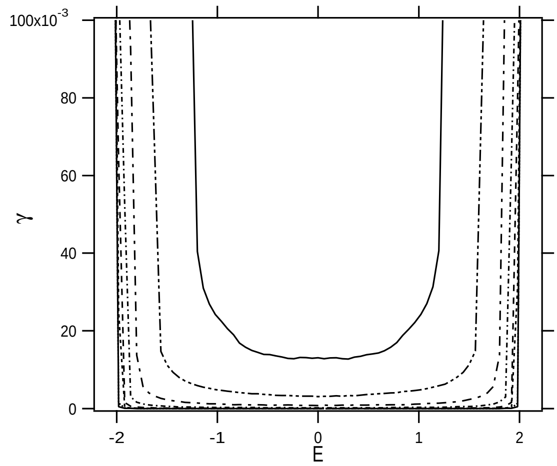

Fig. 1. Lyapunov exponents γ as a function of energy, for several potential strength W within a range [0.01, 0.8]. The system and ensemble size are 2^{20} and 50 respetively. ($B = 2.0$.)

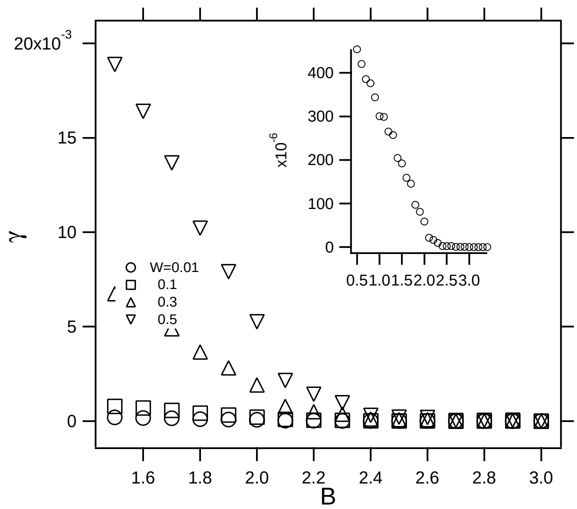

Fig. 2. Lyapunov exponents as a function of bifurcation parameter B, for several potential strength W in symbolic cases ($E = 0$). The inset is expansion of a case, $W = 0.01$.

4. Summary and Discussion

We investigated the localization in 1D disordered system with structural long-range correlation. Numerical results shows a possibility of the presence of the mobility edge for non-stationary regime ($B > 2$). However, we must note that zero Lyapunov exponent does not always mean extended states. Power-law localized state also has zero Lyapunov exponents. It is interesting to investigate the transition dividing exponential localization and non-exponential one such as power-low localization. In 2D disordered system the localization length is much larger than 1D disordered system. Correlation effects on the 2D disordered system is also interesting.[5] The more details will be given in our forthcoming paper.[7]

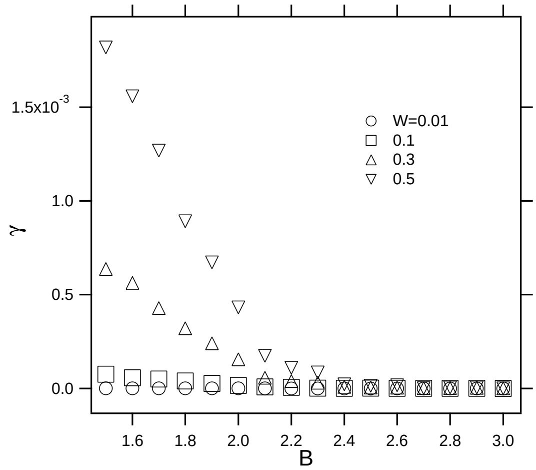

Fig. 3. Lyapunov exponents as a function of bifurcation parameter B, for several potential strength W in non-symbolic cases ($E = 0$).

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