DELOCALIZATION AND DISSIPATIVE PROPERTY IN 1D DISORDERED SYSTEM WITH OSCILLATORY PERTURBATION

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Energy relaxation dynamics in a simple quantum model of electron-phonon system is numerically investigated. We show delocalization in one-dimensional disordered electronic system with coherent harmonic perturbations. The appearance of the delocalization implies that the system has potential for irreversibility and dissipation. Next, we investigate dissipative property of the dynamically delocalized state and show that an irreversible quasistationary energy flow indeed appears in the form of a "heat" flow when we couple the perturbed system with an autonomous mode.

1 Introduction

There have been some attempts examing quantum dissipation [1]. The most popular approaches are heat-bath approach (with infinite number of phonon modes) [2,3], random matrix bath approach [4], linear response theory [5,6], and Landauer formula [7,8,9], and so on. In the orthodox approaches, stochastization mechanisms and heat reservoir consisting of infinite number of degrees of freedom are explicitly or implicitly assumed in advance [10,11,2]. A more interesting scenario of the electronic stochastization is the possibility that the stochastization mechanism is generated in the system *without any help of the time-dependent stochastic source*.

In the present paper, it is demonstrated that contrary to traditional theories infinite number of phonon modes are not necessary and just few phonon modes are sufficient for the delocalization and irreversible energy transfer from the scattered electron to the phonon modes to be induced if the scattering potential is spatially irregular [12,13,14]. In the concrete, a possibly simplest situation is modeled by one-dimensional disordered system (1DDS) coupled with finite number of harmonic time-dependent perturbation and/or harmonic oscillators. Delocalization phenomena and energy relaxation dynamics of the system are numerically investigated. Some of the further details of the numerical results have been reported in references [13,15]. Here we give a short review of the papers and some new results. Note that classically chaotic system with quasi-periodic perturbation have also been used in order to investigate the

localization-delocalization transition by several authors [16,17,18].

The outline is given as follows. In the next section we explain the two kinds of the models we used. In section 3, we give some numerical results about quantum diffusion of initially localized wave packet in 1DDS with oscillatory perturbation. In section 4, we show numerical result about occurrence of dissipative phenomena in closed quantum system, which is an autonomous system, consisting of 1DDS and oscillators. In section 5, we investigate a quantum state of the autonomous mode during occurence of the energy flow, and give a simple phenomenological interpretation for the thermalization phenomena. The last section is devoted to summary and discussion. Derivations of some equations are added in appendices.

2 Model

2.1 Autonomous model

We use 1-DDS coupled with a few harmonic oscillators in order to investigate the energy transfer between the electron and the phonon modes. The total Hamiltonian H_{tot} is consisting of tightly-binding electronic part H_{el}, harmonic oscillators $H_{ph,M}$ with incommensurate frequencies $\{\omega_i\}$ and interaction part H_{int} between them with coupling strength $\{b_j\}$ as follows:

$$H_{tot} = H_{el} + H_{ph} + H_{int}, \tag{1}$$

$$H_{el} = \sum_{n=1}^{N} |n > V(n) < n| + \sum_{n}^{N} (|n >< n+1| + |n+1 >< n|), \tag{2}$$

$$H_{ph,M} = \sum_{j=1}^{M} \left(\frac{\hat{p}_j^2}{2} + \frac{\omega_j^2 \hat{q}_j^2}{2} \right), \tag{3}$$

$$H_{int} = \sum_{n=1}^{N} \sum_{j=1}^{M} (|n > V(n) < n|) b_j \hat{q}_j. \tag{4}$$

$V(n)$ is the onsite energy of electron at the site n, which varies at random in the range $[-W, W]$ from site to site. If the number of phonon mode goes to infinity with an analytical frequency spectrum, then the phonon system becomes a heat reservoir implicitly or explicitly supposed in orthodox theories, but in our treatment the number of phonon modes is *finite*.

2.2 Nonautonomous models

If the harmonic oscillators are highly excited, the model H_{tot} becomes equivalent to a simple time-dependent Hamiltonian perturbed by a classical driving force. Indeed, oscillatory external perturbation $V(n,t) = V(n)[1 + \frac{\epsilon}{\sqrt{L}}\sum_{i=1}^{L}\cos(\Omega_i t)]$ can be mathematically identified with highly excited quantum harmonic oscillators [19,1,12]. Accordingly if the approximation be used for some autonomous phonon modes we can replace them by external perturbation. Then the time-dependent Shrődinger equation that we generally simulate becomes

$$\begin{aligned} i\hbar\frac{\partial\Psi^{tot}(n,\{q_j\},t)}{\partial t} &= \Psi^{tot}(n+1,\{q_j\},t) + \Psi^{tot}(n-1,\{q_j\},t) \\ &+ \{\sum_{j=1}^{M}\left(\frac{\hat{p}_j^2}{2} + \frac{\omega_j^2\hat{q}_j^2}{2}\right) + \sum_{n=1}^{N}\sum_{j=1}^{M} b_j V(n)\hat{q}_j \\ &+ V(n)[1 + \frac{\epsilon}{\sqrt{L}}\sum_{i=1}^{L}\cos(\Omega_i t)]\}\Psi^{tot}(n,\{q_j\},t), \end{aligned} \quad (5)$$

where the $\Psi^{tot}(n,\{q_j\},t)$ represent the wave function of the whole system in a site basis. One of the advantage of this model is that although the number of the autonomous mode M is limited due to computer power, we can freely control the number of the frequency components L of the harmonic perturbation. In the simulation we set M =0 or 1 and/or L =0,1,$\cdots$,5. For convenience, we refer Hamiltonian in eq.(5) as $H_{L,M}^{tot}$ in the following sections

3 Dynamical Delocalization in Nonautonomous System

In our previous paper [13] we showed that the 1DDS exhibits a remarkable delocalization behavior when it is perturbed by classical oscillating forces with several frequency components. The model system is just the model (5) without the harmonic oscillators, i.e., M=0. Such a delocalization phenomenon is a key to understand the occurrence of irreversibility and dissipation.

3.1 Dynamical Delocalization

When oscillatory harmonic perturbations are applied to 1DDS, an initially localized wave packet of electron ($\Psi(t=0) = \delta_{n,0}$) spreads unlimitedly, and we called such a quantum state *dynamically delocalized state*. It is very interesting that such a non-localized state can be easily realized only by applying

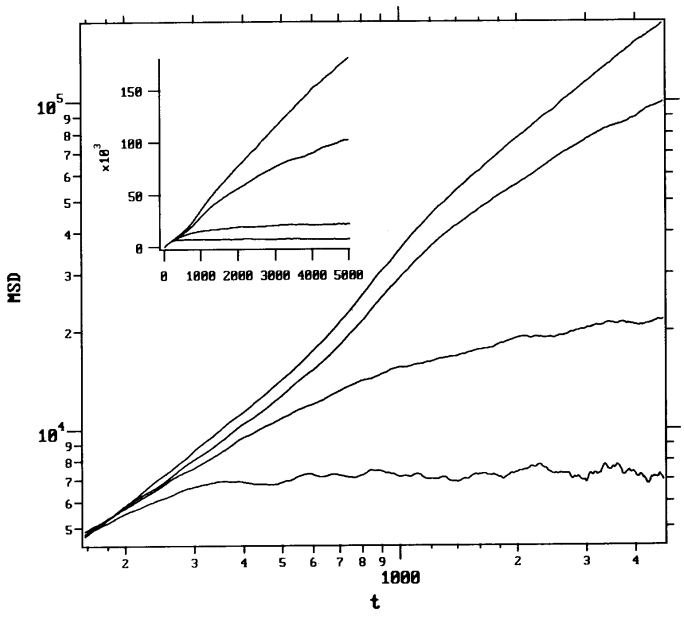

Figure 1. Logarithmic plots of time-dependent MSD of some cases (L =0,1,2,5), where ϵ =0.5, $W = 0.9$ and $\hbar = 1$. 20 different configurations are used for the ensemble average. The insert is real scale one.

an weak coherent perturbation. The delocalization property can be quantitatively characterized by the mean square displacement (MSD) of the wave packet: $m_2(t) =< \Psi(t)|\hat{n}^2|\Psi(t) >$, where $\hat{n} \equiv \sum_{n=1}^{N} n|n >< n|$ is the position operator and $\Psi(t)$ is the time-dependent wave packet.

We performed longer time simulation for larger system than the previous ones [13]. The results are shown in Fig.1 for monochromatically ($L = 1$) and polychromatically ($L \geq 2$) perturbed cases. It is found that the wave packet, which is localized without the interaction with the oscillatory perturbation [20,21], spreads beyond the original localization length as time elapses. The diffusive behavior is observed within the time scale accessible by numerical computations, and the diffusion process is not in general the normal diffusion but a subdiffusion, which is characterized by a power law increase:

$$m_2(t) \sim t^{\alpha}, (0 < \alpha \leq 1). \tag{6}$$

The subdiffusive behavior approach the normal diffusion ($\alpha = 1$) promptly as the number L of the frequency and/or the perturbation strength ϵ increase. However, we note that in the monochromatic case ($L = 1$) the diffusive behavior is suppressed at a certain level which is longer than the original localization length. In the next subsection, we consider further details of the monochromatically perturbed cases ($L = 1$).

The appearance of the diffusive behavior of the wave packet in real space implies that through the dynamic interaction with the coherent perturbation a quantum-mechanical pure state is transformed into a complex pure state which may be called "stochastic" state. The Anderson localized state is thus unstable against weak dynamical perturbations, and a delocalization manifests itself in the form of an unlimited diffusion. Further it has been also revealed that the dynamical delocalization properties obeys remarkable spatio-temporal scaling laws. It were discussed extensively in our previous paper and we do not repeat it in the present paper [13].

3.2 Monochromatically perturbed case (L=1)

We consider the localization in the monochromatically perturbed 1DDS. First, we numerically confirm quasieigenstates of the time-periodic system. The quasieigenstates are defined by eigenstates $\{|\Phi_\alpha >\}$ of one-periodic time-evolution operator $U(T \equiv \frac{2\pi}{\Omega_1})$ as,

$$U(T)|\Phi_\alpha >= \exp(-i\eta_\alpha)|\Phi_\alpha >, \tag{7}$$

$$U(t) = \exp(-\frac{i}{\hbar}\int_0^t H^{tot}_{M=0,L=1}(t)dt) \tag{8}$$

The property of quasieigenstates are directly related to the localization property of the wave packet $\Psi(n, t = kT) =< n|U(T)^k|\Psi(t = 0) >$ in site representation. In Fig.2, we show some typical quasieigenstates $|u_\alpha(n)| = | < n|\Phi_\alpha > |$ for the 1-DDS with monochromatic perturbation. The quasieigenstates are exponentially localized, and it is consistent with appearance of the suppression of the diffusive behavior at certain level in Fig.1. In appendix A, a method in order to calculate numerically the quasieigenstates is given.

Next we consider interpret the localization phenomena by transforming the time-dependent Schrődinger equation to sationary Schrődinger equation. Inserting $\Psi(n, t) = \exp(-i\eta t)\sum_k C_{n,k}\exp(-ik\Omega t)$ to equation (5), the amplitude $C_{n,k}$ obey following equation.

$$\begin{aligned}\hbar\eta C_{n,k} = &(V(n) - \hbar k\Omega)C_{n,k} + (C_{n+1,k} + C_{n-1,k}) \\ &+ \frac{\epsilon V(n)}{2}(C_{n,k+1} + C_{n,k-1}),\end{aligned} \tag{9}$$

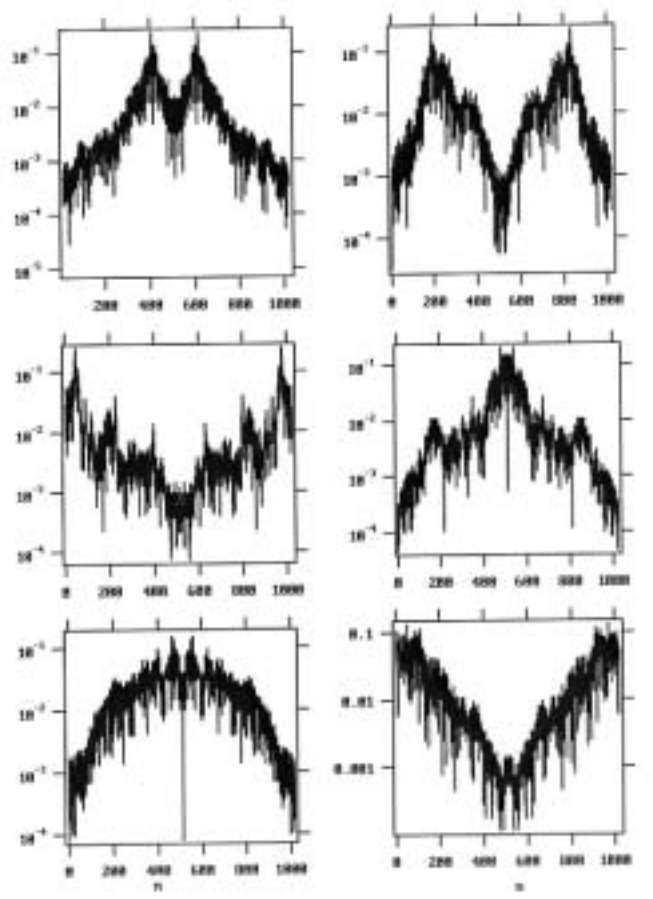

Figure 2. Some quasieigenstates $|u_\alpha(n)| = | < n|\Phi_\alpha > |$ for one-periodic time evolution operator $U(T = 2\pi/\Omega_1)$ of monochromatically perturbed 1DDS in Fig.1.

where $\Omega = \Omega_1$. (See appendix B.) This equation is equivalent to two-dimensional tightly binding system with static electric field and off-diagonal randomness in $k-$direction. Roughly speaking, we found that at least in monochromatically perturbed 1DDS complicated phenomena based on two kinds of the localization, Anderson localization and Stark-ladder localization, are mixed [22,23,24]. It is difficult to analytically get the exact evidence for the

localization because our system is a nonseparable system.

3.3 Energy transfer

The non-autonomous model $H^{tot}_{L,M=0}(t)$ in eq.(5) without the autonomous mode can be transformed into an autonomous version H^{aut}_{L} composed of 1DDS and *linear* oscillators as follows.

$$H^{aut}_{L} = H_{el} + \sum_{j=1}^{L} \Omega_j J_j + H_{osc,L}(\{\phi_j\}), \tag{10}$$

$$H_{osc,L}(\{\phi_j\}) = V(n)\frac{\epsilon}{\sqrt{L}} \sum_{j=1}^{L} \cos\phi_j \tag{11}$$

where $J_j = -i\hbar\frac{\partial}{\partial\phi_j}$. The relation between eigenstates of the autonomous version H^{aut}_{L} and quasieigenstaes of the nonautonomous model $H^{tot}_{L,M=0}(t)$ is given in appendices A and C. It is worth noting that the autonomous model are equivalent to the highly excited harmonic oscillators and works as the source of harmonic perturbation in the nonautonomous model. Based upon the autonomous picture, we can discuss the exchange of energy between the electron and the "phonon modes" of the linear oscillators. The backaction of the delocalization of electron to the phonon modes will results in an excitation and/or deexcitation in the action space of the phonon modes.

We discuss on the exchange of energy between the electronic system and the perturbing system. Since we can introduce an autonomous version of the non-autonomous model, we can explicitly compute the energy which flows from the electronic system to the perturbing mode, which is represented by the expectation value $< \Psi^{aut}(t)|\sum_{j=1}^{L}\Omega_j J_j|\Psi^{aut}(t) > \equiv E_J$, where $\Psi^{aut}(t)$ is the time-dependent wave function of the autonomous system $H^{aut}_{L}(t)$. By using the formula (22) in an appendix C, it immediately follows that

$$E_J(t) = \int_0^t ds < \Psi(s)|\frac{\partial H^{tot}_{L,M=0}(s)}{\partial s}|\Psi(s) >, \tag{12}$$

where $\Psi(t)$ is the time-dependent wavefunction of the nonautonomous system $H^{tot}_{L,M=0}(t)$. The derivation is given in appendix C.

Figure 3 shows the ensemble averaged energy $< E_J(t) >$ transferred to the phonon modes during the time evolution process depicted in Fig.1. It is evident that the phonon energy fluctuates around a certain level, and do not show any signature of net energy transfer between the phonons and the electron. This is because the backaction of the electron makes the phonons

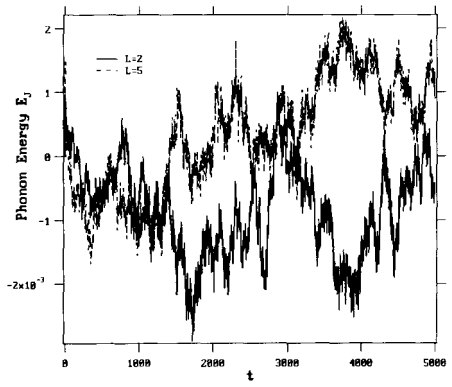

Figure 3. Time-dependence of ansemble-averaged phonon energy $< E_J(t) >$ in polychromatically perturbed cases ($L = 2, 5$). The parameters are same values to Fig.1.

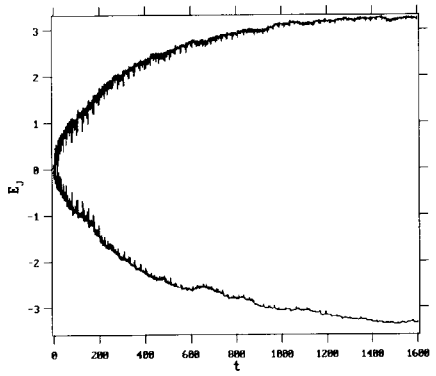

Figure 4. Time-dependence of phonon energy $E_J(t)$ of the nonautonomous modes in polychromatically perturbed case ($L = 3$). The initial states of the electron are set in eigenstates at high energy ($n_{el} = 3$) and low energy ($n_{el} = 123$), where n_{el} is number of energy level from the top of the energy level. The other parameters are same values to Fig.1.

excite and deexcite symmetrically around the initial Fock state. In other words, the phonons show a diffusive motion around the initial state along the ladder of Fock basis.

Another important reason is that the initial electronic state ($\Psi(t=0) = \delta_{n,0}$) is a mixture of almost all the localized basis with negative and positive energy eigenvalues, and the expectation value of energy is close to zero. In other words, the electron has no excessive energy. Indeed, if we choose the initial electronic state to the highest-energy localized state (or the lowest-energy localized state), the symmetry of the diffusion of phonons around the initial phononic state is broken and the phonon absorbs (or emits) energy as is depicted in Fig.4, although the net energy flow is not very intensive because the excited diffusion flow and the deexcited diffusion flow almost cancel out each other. This fact implies that, if a phonon is prepared initially not in a highly excited state but in the ground states of the harmonic oscillators, the deexcitation is forbidden, and the phonon mode will be excited diffusely toward the higher Fock states, which means the onset of a one-way energy transport from the electronic system to the phonon system. This is just the main subject of the subsequent sections.

4 Energy Relaxation of Delocalized States

As has been seen in last section, the additional monochromatic perturbation enhances the localization length of 1DDS, and polychromatic perturbation makes the localization length diverge. In this section we observe the energy relaxation of the 1DDS perturbed by coherently oscillatory force. In the concrete, we couple the dynamically perturbed 1DDS with a harmonic oscillator in order to investigate energy flow between electron and the autonomous mode. We prepare the electron initially in a sufficiently high excited eigenstate and set the autonomous phonon in the ground state, and compute the time-dependent phononic energy $E_{ph}(t)$, electronic energy $E_{el}(t)$, and MSD of the electron.

4.1 Monochromatically perturbed case ($M = 1, L = 1$)

As shown in Fig.5, if the coupling strength is large enough ($b = 1.0$) the diffusion and a one-way energy transfer continues until it reaches a fully relaxed state even at smaller values of perturbation strength ϵ. The behavior is quite different from the non-perturbed case ($L = 0$) which it saturates at certain level on the way of the fully relaxation [13]. It seems that at early stage of the time-evolution the phonon energy exhibits a nice linear increase, while the electronic energy decreases monotonously until it reaches to almost zero energy level. The flowing rate of energy increases in accordance with the increment in the perturbation strength ϵ. The MSD of the electronic state also

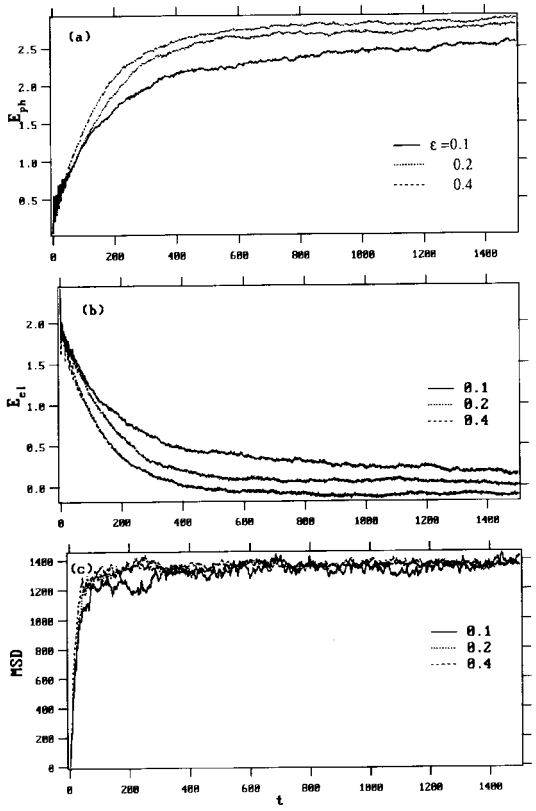

Figure 5. Time-dependence of (a) an electronic energy, (b) a phononic energy and (c) MSD of electron in monochromatically perturbed cases ($M = 1$, $L = 1$), where $W = 0.9$, $\hbar = 1/8$, $b = 1.0$, $\omega = 0.8$, $\Omega_1 = \sqrt{2}$ and the varius perturbation strength $\epsilon = 0.1, 0.2, 0.4$.

approaches the maximum length allowed by the finite system size. The final electronic state with almost zero energy can be regarded as an equilibrium state which contains all the localization basis, whose energies are distributed symmetrically around zero, with even statistical weight. We can judge that the system is delocalized and becomes *completely dissipative* in such a coupling strength regime.

Furthermore, we show some cases with different electronic initial states in Fig.6. The number of the eigenstates n_{el} =4, 14 and 21 from the top of the

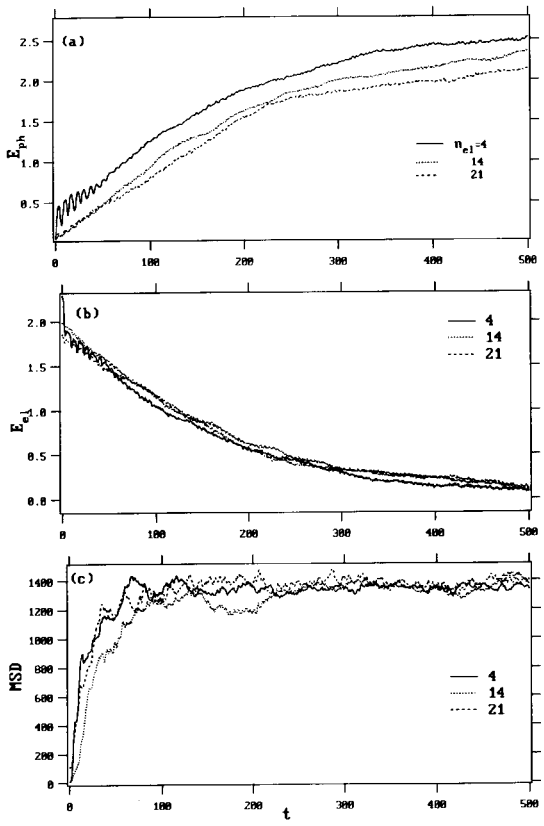

Figure 6. Time-dependence of (a) an electronic energy, (b) a phononic energy and (c) MSD of electron in monochromatically perturbed cases ($M = 1$, $L = 1$), where $W = 0.9$, $\hbar = 1/8$, $b = 1.0$, $\omega = 0.8$, $\epsilon = 0.4$ and the frequency Ω_1=$\sqrt{2}$, for three different initial excited eigenstate of electron (n_{el} =4, 14 and 30).

energy level are used as the initial excited states. The other parameters are set the same to the case of the Fig.5. The behavior of the energy relaxation is almost similar to the other case in Fig.5.

As a result, it seems that a stationary energy transport continue slowly for long time before the spread of the wave packet reach the saturation level when the coupling strength is large to make mixing in the system.

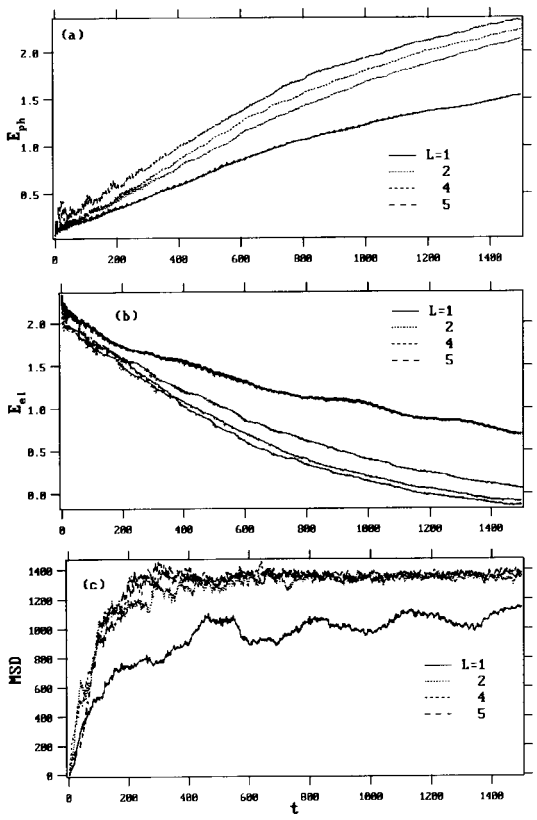

Figure 7. Time-dependence of (a) an electronic energy, (b) a phononic energy and (c) MSD of electron in monochromatically and polychromatically perturbed cases ($M = 1$, $L = 1, 2, 4, 5$), where $W = 0.9$, $\hbar = 1/8$, $b = 0.4$, $\omega = 0.8$ and $\epsilon = 0.4$. The frequency components of the perturbation $\{\Omega_i\}$ are chosen within a range $[0.5, 1.5]$ randomly.

4.2 Polychromatically perturbed case ($M = 1, L \geq 2$)

When the number of the frequency components of the perturbation is larger than or equal to two ($L \geq 2$), the 1DDS exhibits typical symptom of dynamical delocalization. In this subsection we examine dissipative property for the polychromatically perturbed 1DDS.

Typical examples of time-dependent energy transfer between a polychro-

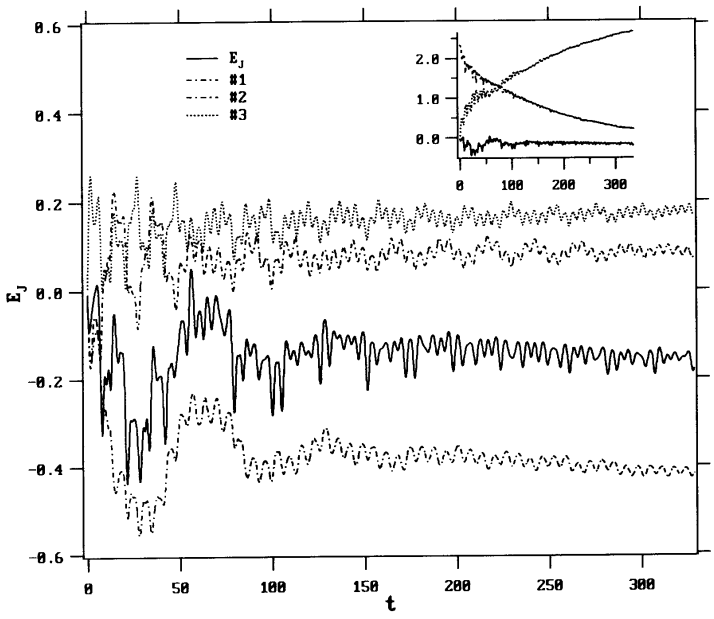

Figure 8. Time-dependence of energy E_J of the total nonautonomous modes and each of the modes $(\#1, \#2, \#3)$ in polychromatically perturbed cases ($L = 3$), where $W = 0.9$, $\hbar = 1/8$, $b = 0.8$, $\omega = 0.8$ and $\epsilon = 0.4$. The frequency components of the perturbation $\{\Omega_i\}$ are chosen within a range $[0.5, 1.5]$ randomly. The insert shows the E_{el}, E_{ph}, E_J in the time evolution.

matically perturbed 1DDS ($L = 2, 4, 5$) and the autonomous mode are depicted in Fig.7. In all cases the MSD grows up to the maximum scale and a complete delocalization is achieved, and the electronic energy shows a very nice relaxation behavior even in the cases with small coupling strength. In the early stage of time-evolution, the electron loses its energy linearly in time. In such a quasi-stationary regime the emission rate of energy per unit time can be well defined [15]. Monotonic increase of phonon energy continues until the wave packet spreads over the system size and the electronic energy approaches to zero level, which indicate a complete delocalization. We can confirm the energy fluctuation in the nonautonomous modes during the energy transfer from electron to the autonomous mode. As shown in Fig.8, energy of each of the nonautonomous modes $E_{J_k}(t)$ fluctuates around certain level due to

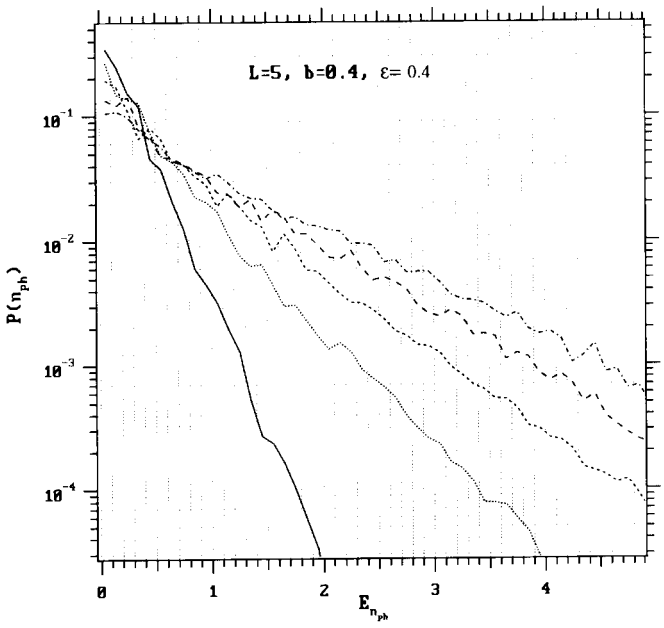

Figure 9. Phonon distribution $P(E_{n_{ph}}) = | < n_{ph}|\Psi(t) > |^2$ v.s. $E_{n_{ph}}$ at several time ($t = 100, 200, 300, 400, 500$) in the polychromatically perturbed case ($M = 1$, $L = 5$). The parameters are $b = 0.4$ and $\epsilon = 0.4$.

exchange of the energy between the each of the modes and electron.

In conclusion, all the above features indicate that a complete dissipation is realized in case of $L \geq 2$.

5 Quantum State of Autonomous Mode

In this section, we pay attention to the quantum state of the autonomous phonon mode during the (quasistationary) energy flow. We show the phonon distribution in polychromatically perturbed case ($L = 5$) as a typical example. Figure 9 shows the semi-log plots of the probability distribution $P(E_{n_{ph}}) = | < n_{ph}|\Psi^{tot}(t) > |^2$ of the autonomous phonon mode as a function of the energy $E_{n_{ph}}$. The Boltzmann-type distribution appears only when quasistationary energy transfer from electron to phonons is observed. As a

result, the phonon mode reaches promptly a "*thermalized state*" characterized by a well defined time-dependent temperature $T(t)$.

A simple phenomenological interpretation is possible for the appearance of the Boltzmann-type distribution as we use a harmonic oscillator. We express the total Hamiltonian in Fock space.

$$\begin{aligned} H^{tot}_{L,M} &= \sum_{n=1}^{N} V(n,t) b^{\dagger}_n b_n + \sum_n (b^{\dagger}_{n-1} b_n + b^{\dagger}_{n+1} b_n) \\ &+ \sum_{j=1}^{M} (\frac{1}{2} + a^{\dagger}_j a_j) \hbar \omega_j \\ &+ b \sum_{n=1}^{N} \sum_{j=1}^{M} \sqrt{\frac{\hbar}{2\Omega_j}} V(n,t) b^{\dagger}_n b_n (a^{\dagger}_j + a_j), \end{aligned} \tag{13}$$

where the b denotes the coupling strength. The $b^{\dagger}_n, b_n$ are creation and annihilation operators of an electron at $n-$th site in real space, which satisfy the usual anticommutation relations for Fermions, $[b_n, b^{\dagger}_m]_+ = \delta_{n,m}$. The $a^{\dagger}_j, a_j$ are creation and annihilation operators for $j-$th energy eigenstate of the autonomous mode. The Heisenberg equation for creation operator of the autonomous mode become following linear equation.

$$\frac{da^{\dagger}_j(t)}{dt} = i\omega_j a^{\dagger}_j(t) + ib\sqrt{\frac{\hbar}{2\Omega_j}} \sum_n U^{\dagger}(t) b^{\dagger}_n V(n,t) b_n U(t). \tag{14}$$

Here we assume some statistical property of the second term of RHS, $R(t) \equiv ib\sqrt{\frac{\hbar}{2\Omega_j}} \sum_n U^{\dagger}(t) b^{\dagger}_n V(n,t) b_n U(t)$. Neglecting the weak dependence of the correlation function $< R^{\dagger}(t_2) R(t_1) > \equiv G(t_1, t_2 - t_1)$ $(t_2 > t_1)$ on t_1 and rapid decay for time region, $t_2 - t_1 > t_c$, the expectation value of the autonomous mode increases in proportion to time. If $a^{\dagger}_j$ is an integration over the stochastic source $R(t)$ with the very short characteristic time t_c. The amplitude $a^{\dagger}_j$ is a sum over statistically independent quantities and hence should obey a Gaussian stochastic process. Regarding $a^{\dagger}_j$ as c-number, the distribution function of $a^{\dagger}_j$ should be the Gaussian distribution $P(a_j, a^{\dagger}_j) \propto \exp\{-\text{const} \times |a_j|^2\}$, which is equivalent to the Boltzmann-type distribution. Moreover, if the Heisenberg equation for density operator of the electron,

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H^{tot}_{L,M}, \rho], \tag{15}$$

can be effectively transformed into a diffusion-type equation, the MSD $< n^2 > = \sum n^2 < n|\rho|n >$ increases linearly in time.

6 Summary and Discussion

Diffusive and dissipative property of 1DDS perturbed by time-dependent harmonic driving force is numerically investigated. We have shown that the 1DDS is sensitive to a coupling with other degrees of freedom and that the 1-DDS driven by periodically time-varying perturbation exhibits a subdiffusive behavior.

We investigated dissipative property of the dynamically delocalized states by coupling the system with an another simple system prepared in the ground state. An irreversible flow of energy from electron to phonons is induced spontaneously even though the number of phonon modes is only two. Electron scattered by an irregular potential emits its energy to the phonon modes, and moreover the phonons are excited to a thermal state characterized by a well defined temperature. In this report, we did not show the results in stochastically perturbed cases, the results are almost similar to that of polychromatically perturbed cases with more than four colors.

Spatial irregularity also plays a crucial role as an origin of quantum irreversibility when it is combined with a dynamical interference arising from some other degrees of freedom. Such a mechanism may provide a simple dynamical modeling to understand the origin of resistivity in solid state materials [25,26].

Note that there are, recently, some interesting reports concerning roles of the chaotic system as a "heat bath" in quantum system with small degrees of freedom [27,28,29,30].

Acknowledgments

This study is based on the collaboration with Professor K.S.Ikeda. Author would like to thank him for valuable suggestions and stimulating discussion. Author appreciates Professor S.Tasaki for inviting me to Japan-Italy Joint Waseda Workshop on: Fundamental Problems in Quantum Physics. Author also thank the participants of the workshop for useful comments and discussion.

Appendix

A Relation between quasieigenstates and eigenstates

In this appendix we show a relation between eigenstates of the autonomous version $H^{aut}_{L=1}$ and Floquetstates of the nonautonomous system $H^{tot}_{L=1,M=0}$. Let us consider eigenvalue equation of time-evolution operator $U(t)$ for one-period

$T = 2\pi/\Omega$,

$$U(T)\phi_\alpha(n) = \exp(-i\eta_\alpha)\phi_\alpha(n), \tag{16}$$

where η_α and $\phi_\alpha(n)$ are αth quasieigenenergy and quasieigenstate in site $n-$representation, respectively. To get the quasieigenstates numerically, we diagonalize matrix $< \ell|U(T)|m >$ which is created by one-periodic time evolution for each of the unit vectors, where the $|m >$ denotes unit vector with nonzero element only in the $m-$th site as $|m >= (|0, 0, \cdots, 1, \cdots, 0 >)^\dagger$.

We define a function $\phi_{\alpha,\ell}(\theta, n)$ by using the operator, the quasieigenstate and quasieigen energy,

$$\phi_{\alpha,\ell}(\theta, n) \equiv \exp(-i\eta_\alpha\theta/\Omega + i\ell\theta)U(\theta/\Omega)\phi_\alpha(n), \tag{17}$$

where ℓ represent arbitrary integer. It can be found that this function becomes the eigenstate of an autonomous version $H^{aut}_{L=1}$ of the monochromatically perturbed Hamiltonian $H^{tot}_{L=1,M}(t)$, by inserting the function into the eigenvalue equation, $H^{aut}_{L=1}\phi_{\alpha,\ell}(\theta, n) = E_{\alpha,\ell}\phi_{\alpha,\ell}(\theta, n)$. As a result we see that the ℓ means a quantum number which characterize the eigenstate of the linear oscillator, and the eigenenergies of the autonomous version are given by,

$$E_{\alpha,\ell} = \eta_\alpha + 2\pi\ell. \tag{18}$$

B Floquet States

We derive a stationary Shrődinger equation by inserting $\Psi^{tot}(n, t)$ of eq.(19) in the time-dependent Schrődinger equation (5) for $H^{tot}_{L,M=0}$.

$$\Psi^{tot}(n, t) = \exp(-i\eta t) \sum_{k_1,k_2,\cdots,k_L} C_{n,k_1,k_2,\cdots,k_L} \exp(-i\sum_{j=1}^{L} k_j\Omega_j t), \tag{19}$$

The amplitude $C_{n,k_1,k_2,\cdots,k_L}$ obey following equation.

$$\begin{aligned}
\hbar\eta C_{n,k_1,k_2,\cdots,k_L} &= (V(n) - \hbar\sum_{j=1}^{L} k_j\Omega_j)C_{n,k_1,k_2,\cdots,k_L} \\
&\quad + (C_{n+1,k_1,k_2,\cdots,k_L} + C_{n-1,k_1,k_2,\cdots,k_L}) \\
&\quad + \frac{\epsilon V(n)}{2\sqrt{L}} \quad \{C_{n,k_1+1,k_2,\cdots,k_L} + C_{n,k_1-1,k_2,\cdots,k_L} \\
&\quad + C_{n,k_1,k_2+1,\cdots,k_L} + C_{n,k_1,k_2-1,\cdots,k_L} \\
&\quad + \cdots \\
&\quad + C_{n,k_1,k_2,\cdots,k_L+1} + C_{n,k_1,k_2,\cdots,k_L-1}\},
\end{aligned} \tag{20}$$

It can be regarded as $(L+1)-$dimensional tight-binding system with disorder under external field.

C Derivation of eq.(12)

In this appendix we derive the expression of the phonoic energy in the nonautonamous system. We show the expression only in monochromatically perturbed case. Extension to polychromatically perturbed cases is easy. First we consider following autonomous Hamiltonian:

$$H^{aut}_{L=1} = H_{el} + H^{osc}_{L=1}(\phi) + \Omega J. \tag{21}$$

The term $\Omega J (\equiv H_J = -i\hbar\Omega\frac{\partial}{\partial\phi})$ represents a linear oscillator, where J and Ω are action variable and frequency of the motion, respectively. The ϕ is angle variables which is conjugate to J. The time evolution operator is given as follows:

$$\begin{aligned} U^{aut} &\equiv \exp(-i\frac{H^{aut}t}{\hbar}) \\ &= \exp(-i\frac{\Omega Jt}{\hbar})\hat{T}\exp\big(-\frac{i}{\hbar}\int_0^t ds\{H_{el} + H^{osc}(\Omega s + \phi)\}\big). \end{aligned} \tag{22}$$

where $\hat{T}$ is time-ordering operator.

We set an initial state,

$$|\Psi^{tot}(t=0)> = |i> \otimes |\phi_0>, \tag{23}$$

where the $|i>$ and $|\phi_0>$ are initial state of electron and eigenstate of the phase operator, respectively. Here we consider a derivative of $<\Psi^{tot}(t)|H_J|\Psi^{tot}(t)>$ by time,

$$\begin{aligned} \frac{d<\Psi^{tot}(t)|H_J|\Psi^{tot}(t)>}{dt} &= \frac{i}{\hbar} <\Psi^{tot}(t)|[H^{aut}_{L=1}, H_J]|\Psi^{tot}(t)> \\ &= -<\Psi^{tot}(t)|\Omega\frac{\partial H^{osc}_{L=1}(\phi)}{\partial\phi}|\Psi^{tot}(t)> \\ &= -\Omega <\phi_0|\otimes <i|U(t)^\dagger \exp(\frac{iJ\Omega t}{\hbar})\frac{\partial H^{osc}_{L=1}(\phi)}{\partial\phi} \\ &\times \exp(\frac{-iJ\Omega t}{\hbar})U(t)|i> \otimes |\phi_0> \\ &= -<i|U^\dagger(t)\frac{\partial H^{osc}_{L=1}(\Omega t+\phi)}{\partial t}U(t)|i>. \end{aligned} \tag{24}$$

In the last equal sign we used following relations.

$$\exp(\frac{iJ\Omega t}{\hbar})\frac{\partial f(\phi)}{\partial \phi}\exp(\frac{-iJ\Omega t}{\hbar}) = \frac{\partial f(\phi+\Omega t)}{\partial \phi} = \frac{1}{\Omega}\frac{\partial f(\phi+\Omega t)}{\partial t}. \quad (25)$$

As a result, we can get integration of above equation (24), i.e., $E_J(t)$, by calculating of the time-evolution of the initial state $|i>$ of the electron.

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